

on target

luametatex & context lmtx

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1 Introduction

This is the seventh wrapup of the LuaTeX and LuaMetaTeX development cycle. It is dedicated to all those users who kept up with developments and are always willing to test the new features. Without them a project like this would not be possible.

At the time this introduction is written the LuaMetaTeX code base is rather stable and quite a bit of the MkIV code base has been adapted to new situation. But, as usual, there are always new possibilities to explore, so I expect that this document will grow over time as did the others. I'm not going to repeat all that has been done because that's what the previous episodes are about.

As the title suggest, we're still on target. When the LuaMetaTeX project started there actually was no deadline formulated so in fact we're always on target. The core components TeX, MetaPost, and Lua are all long term efforts so we're in no hurry at all. However, this is the year that a fast pace will become a slow pace with respect to the LuaMetaTeX code base. There are still some things on the agenda but in principle the goals are reached. One problem in today's code development is that useability and quality seems to relate to the amount of changes in code. No update can mean old, unusable and uninteresting. It's probably why some sources get this silly yearly copyright year update. However, the update cycle of good old TeX has an decade interval by now while it is still a pretty useable program. It would be nice to end up in such a long term cycle with LuaMetaTeX: bug fixes only.

Although ConTeXt has always adapted early to new developments (color, graphics, pdf, MetaPost, -TeX, pdfTeX, LuaTeX, utf, fonts) the effects on the ConTeXt code base are mostly hidden for users. There have been some changes between MkII and MkIV, simply because there has been a shift from specific eight bit encodings to utf and Type1 to OpenType fonts. Both had an impact on important subsystems: input encodings, font definitions and features, language and script support. On other subsystems the impact was hardly noticeable, like for instance backend related features (these have always been kind of abstract). That doesn't mean that these haven't changed deep down, they definitely have. Some mechanisms became better in MkIV, simply because less hackery was needed. My experience is that when users see that it gets better or easier, they are also willing to adapt the few lines in their document source that benefit from it. Of course the impact on the MetaPost integration in ConTeXt had a real large impact, especially in terms of performance.

The upgrade to LMTX, the version of ConTeXt for LuaMetaTeX, is even less visible although already some new mechanisms showed up. This time a couple of engine specific features have been improved and made more flexible. In fact, the whole code base of the engine has been overhauled. This happened stepwise because we had to make sure all things kept working. As a first step code was made independent of the compilation infrastructure and the dependencies, other than a very few small ones, have been removed. The result is a rather lean and mean setup, even when we consider what has been added at the primitive level and traditional subsystems. A benefit is that in the meantime the LuaMetaTeX LMTX combination outperforms LuaTeX with MkIV, something that was not ensured when the built-in pdf backend was removed and delegated to Lua. By binding development closely to ConTeXt we also hope that the code base stays clean of arbitrary extensions.

Because in the end, TeX is also a programming language, there have been extensions that make programming easier. There is already a stable middle layer of auxiliary macros in ConTeXt that help the user who likes to program but doesn't like real low level primitives and dirty tricks, but by extending the primitive repertoire a bit users can now stay closer to the original TeX concepts. Adding more and more layers of indirectness makes no sense if we can improve the bottom programming layer. It also makes coding a bit more natural (the TeX look), apart from offering performance benefits. This is where you can see differences between the MkIV and LMTX code base which for that reason is now nearly split completely. The

MetaPost subsystem has been extended with proper scanners so that we can enhance the interfaces in a natural way and as a result we also have an upgraded code base there. We also moved to Lua5.4 and will keep up as long as compatibility is no issue. Some Lua code is likely to remain common between MkIV and LMTX, for instance font handling and helpers but we'll see where that ends.

The LuaMetaT_EX engine provides control over most internals and there are all kind of new interesting features. Decades of ConT_EXt development are behind that. Also, in the days that there were discussions about extending T_EX, ConT_EXt was not that much of influence and on the road to and from user group meetings, Taco and I often discussed what we'd like to see added (and some was actually implemented in `eeTeX` but that only lived on our machines. One can consider LuaT_EX to be a follow up on that, and LuaMetaT_EX in turn follows up on that project, which we both liked doing a lot. In some way LuaT_EX lowered the boundary for implementing some of the more intrusive extensions in LuaMetaT_EX and the follow up on mplib. And once you start along that road small steps become large steps and one can as well be try to be as complete as possible. We've come a long way but eventually arrived at the destination. Personally I think we got there by not being in a hurry.

But even targets that are reached can eventually move,

Hans Hagen
Hasselt NL
August 2021⁺⁺

2 Eventually 1.0

2.1 Reflection

This is just a short reflection on how we came to version 1.0 of LuaMetaT_EX. Much has already been said in articles and history documents. There is nothing in here that is new but I just occasionally like to wrap up the current state. At the time of writing, which happens to be the ConT_EXt 2021 meeting, we're somewhere between 0.9 and 1.0 and as usual it reflects a current state of mind.

2.2 Introduction

The development on LuaMetaT_EX took a bit more time than I had in planned when I started with it. I presume that it also relates to the way the T_EX program is looked at: a finished program that converges to a bugless state. But, with version 1.0 near by it makes sense to reflect on the process. Before I go into details I want to remark that when I wrote ConT_EXt I looked at this program from the macro end. I had no real reason to look into the code, and figuring out what happens in a black box is a challenge (and kind of game) in itself. At the time I started using T_EX I had done my share of complex and relatively large scale programming in Pascal and Modula so it's not that I was afraid of languages. It was before the Internet took off and not being in academia and connected one had to figure things out anyway. I did have Don's 5 volume T_EX series but stuck to the T_EX book. Being on msdos I couldn't compile the program anyway, definitely not without the source at hand. I did read the first chapters of the MetaFont book, but apart from being intrigued by it, it was not before I ran into MetaPost that knowing that language took off. Of course I had browsed T_EX the program but not in a systematic way.

I was involved with pdfT_EX development but stayed at my end of the line: needs, applications, testing and suggestions. With LuaT_EX that line got crossed, triggered by the Lua interfaces, but while I focussed on the T_EX end, Taco did the C, and we had pleasant and intense daily discussion on how to move forward. I could not get away any longer with the abstraction but had to deal with nodes and such, which was okay as we were hit the boundaries of convenience programming solutions in ConT_EXt.

When we started our LuaT_EX journey the T_EX follow-up most widely used, pdfT_EX, did have some `-TEX` extensions but in retrospect only a few of those were of relevance to us, like the concept of `\protected` macros¹ and the larger set of registers. And the `-TEX` project, in spite of occasional discussions, never became a continuous effort. The `nts` project that was related to `-TEX` and had as objective an extensible successor produced a Java implementation but that one was never useful (as a starter, its performance was such that it could not be used) and I didn't really look forward to spending time on Java anyway. Taco and I played with an extended `-TEX` but lack of time made that one end up in the archive.

There were some programmatic additions to pdfT_EX but it's main attributes were protrusion, expansion and a pdf backend (Hàn Thế Thành's thesis subject). Features like position tracking were handy but basically just a built-in variant of a concept we already had come up with at the dvi level (using a postprocessing script that later became `dvipos`). There was Omega with a directional model but this engine was always more of an academic project, not a production system.² It was X_YT_EX that moved the T_EX world into the Unicode domain and opened the engine up to new font technologies. Although utf8 was already doable in earlier engines (which is why ConT_EXt used it already for some internals), native support was way more convenient.

¹ In ConT_EXt we always had a protection mechanism and from the LuaT_EX source I learned that the macro bases solution was basically the same as the one used in the engine.

² Aleph was more reliable but never took off, if only because pdfT_EX had a backend.

It was clear that if we wanted to move on we had to make more fundamental steps, but in such a way that it still fit in with what people expect from $\text{T}_{\text{E}}\text{X}$. While it started as a playground by embedding the Lua interpreter, it quickly became clear that we could open up the internals in fundamental ways, thereby also getting around the discussion about to what extent $\text{T}_{\text{E}}\text{X}$ could and should be extended: that discussion could be and was postponed by the opening up. Because we already foresaw some of possibilities it was decided to freeze $\text{ConT}_{\text{E}}\text{Xt}$ for the older engines. It was around the first $\text{ConT}_{\text{E}}\text{Xt}$ meeting that the MkII and MkIV tags showed up, around the same time that $\text{LuaT}_{\text{E}}\text{X}$ became useable. More than a decade later, when $\text{LuaT}_{\text{E}}\text{X}$ basically had become frozen, at another meeting it was decided to move on with $\text{LuaMetaT}_{\text{E}}\text{X}$: the $\text{LuaT}_{\text{E}}\text{X}$ project was pretty much a $\text{ConT}_{\text{E}}\text{Xt}$ project and that follow up would be even more driven by $\text{ConT}_{\text{E}}\text{Xt}$ users and usage. But how does it all feel 15 years later? I'll try to summarize that below. It will also explain why I got more audacious in extending the $\text{LuaT}_{\text{E}}\text{X}$ engine into what is now $\text{LuaMetaT}_{\text{E}}\text{X}$. This also related to the fact that at some point I realized that progress just demands taking decisions, and it happens that we can make these in the perspective of $\text{ConT}_{\text{E}}\text{Xt}$ without side effects for other $\text{T}_{\text{E}}\text{X}$ usage. It is also fun to experiment.

2.3 Extending necessary parts

The $\text{pdfT}_{\text{E}}\text{X}$ program, having a backend built in already supports the usage of wide TrueType but it was $\text{X}_{\text{F}}\text{T}_{\text{E}}\text{X}$ that first provided using them directly in the frontend. But that happened within the concept of traditional $\text{T}_{\text{E}}\text{X}$, especially when it comes to math. There are some extra primitives to deal with scripts and languages but (and this is personally) I decided that these didn't really fit in the way $\text{ConT}_{\text{E}}\text{Xt}$ looks at things so MkII doesn't support anything beyond the fonts. The $\text{X}_{\text{F}}\text{T}_{\text{E}}\text{X}$ program first was available on Apple computers and font support was closely related to its technology as well as technologies that relate to where the program originates. Later other operating systems became supported too.

We decided in $\text{LuaT}_{\text{E}}\text{X}$ to delegate 'everything fonts' to Lua, for a good reason: we didn't want to be platform dependent. And using libraries has the danger of periodical enforced fundamental changes because in these times software politics and fashion have short cycles. The fact that $\text{X}_{\text{F}}\text{T}_{\text{E}}\text{X}$ later changed the font engine proved that this was a good decision. At some point $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$ decided to use a special version of $\text{LuaT}_{\text{E}}\text{X}$ that uses a font library as alternative, which is fine, but that also introduces a dependency (and frequent updating of the binary). The $\text{LuaT}_{\text{E}}\text{X}$ engine has a slim variant of the FontForge library built in for reading various font formats and its backend can embed subsets of OpenType, Type1 and traditional bitmap fonts. At some point $\text{ConT}_{\text{E}}\text{Xt}$ switched to its own Lua based font file interpreter and experimented with a Lua based backend that later became exclusive for $\text{LuaMetaT}_{\text{E}}\text{X}$. It became clear that we could do with less code in the engine and thereby less dependencies.

In this perspective it is also good to notice that the $\text{LuaT}_{\text{E}}\text{X}$ engine has no real concept of Unicode: it just expects utf8 and that's it. All internals provide enough granularity to support Unicode. The rest has to come from the macro package, as we know that each one does it its own way. There are no dependencies on Unicode libraries. You only have to look at what ends up on your system when you install a program that just juggles bytes to notice that by including one library a whole lot gets drawn in, most of which is not relevant to the program and we don't want that. It might start small but who knows where one ends up. If we want users to be able to compile the program, we don't want to end up in dependency hell.

The $\text{LuaT}_{\text{E}}\text{X}$ project was, apart from curiosity and potential usage in $\text{ConT}_{\text{E}}\text{Xt}$, initially also driven by the Oriental $\text{T}_{\text{E}}\text{X}$ project that aimed at high quality bidirectional typesetting. There the focus was on fonts as well as processing paragraphs. That triggered all kinds of opening up of internals and once $\text{ConT}_{\text{E}}\text{Xt}$ started swapping (and adding) mechanisms using Lua more came to fruit. In the end it took a decade to reach version 1.0 and we could have stopped there knowing that we're quite prepared for the future.

Although the whole $\text{T}_{\text{E}}\text{X}$ concept didn't change, there were some fundamental changes. From the documentation by Don Knuth it becomes clear that interpreting is closely interwoven with typesetting: the so called

main interpretation loop calls out to font processing, ligature building, hyphenation, kerning, breaking lines, processing pages, etc. In LuaTeX these steps became more independent simply because the processing of fonts (via Lua) came down to feeding a linked list of nodes to a callback function. That list should be hyphenated if needed (a now separated step) and if needed the traditional font processing could be applied (ligature building and kerning). But, although one can say that we already got away from the way TeX works internally, most documentation to the original program still applied, simply because the fundamental approach was the same. We didn't feel too guilty about it and I don't think anyone objected. By the way, the same is true for the math subsystem: we had to adapt it to OpenType parameters and formula construction and although that was inspired by TeX it definitely was different, even to the extent that the math fonts that evolved in the community are now a strange hybrid of old and new.

2.4 Getting around the frozen machinery

So why did the LuaMetaTeX project started at all? There has been plenty written on how LuaTeX evolved and the same is true for LuaMetaTeX so I'm not going to repeat that here. It is enough to know that the demand for a stable and frozen LuaTeX by other users than ConTeXt simply doesn't go well with further experiments and we still had plenty ideas. Because at some point Taco had no time I was already responsible for quite some additions to the LuaTeX program so it was no big deal to switch to a an even more extensive mix of working with “TeX the macro language” and “TeX the program”.

The first priorities were with some basic cleanup: remove unused font code, get rid of some ever changing libraries and remove the backend related code. I could do that because I already had a Lua driven backend in MkIV (which was removed later on) and font handling was already all done in Lua. The idea was to go lean and mean, and indeed, even with all kind of extensions, the binary is much smaller than its predecessor, which is nice because it is also a Lua engine. Simplifying the build so that users can easily compile themselves was also of high priority because I considered the rather large and complex setup as a time bomb. And I also had my doubts if we could prevent the LuaTeX engine to evolve over time in a way that made it less useable for ConTeXt.

But, interestingly all this extending and pruning didn't feel like I was violating the concept of a long term stable engine. In fact, original TeX has no backend either, just a simple binary serialization of output (dvi). And by removing some font related frontend code we actually came closer to the original. I suppose that these decisions slowly made me aware of the fact that there was no reason to not consider more drastic extensions. After all, wasn't the `-TeX` project also about extending.³

When we look at LuaMetaTeX 1.0 we still see the expected machinery there but many subsystems have been extended. Once I made the decision that it's now or never, each subsystem got evaluated against my long term wish list and usage in ConTeXt. Now, let's be clear: I basically can do all I want in LuaTeX but that doesn't mean it's always a pretty solution. And to make the ConTeXt code base better to understand for users, even if it is already rather consistent and set up to be readable, is one of my objectives. I spend a lot of time on readability: I cannot stand a bad looking source and over time the look and feel is also determined by the way the ConTeXt interfaces and related syntax highlighting evolved, especially the TeX, MetaPost, Lua mix. This is why LuaMetaTeX has some extensions to the macro language.

So, while some might argue that “It can already be done.” I decided to ignore that argument when the actual solutions came too close to “See how well I can do this using dirty tricks!”. If we can do better, without harming the system, let's do it: Lua did it, C did it and even Don Knuth switched from Pascal to C. If we want we can put all the extensions under the “TeX is meant to be extended” umbrella, as long as we call it different,

³ Although non of the ideas that Taco and I discussed on our numerous trips to meetings all over the world ever made it into that engine.

which is what we do. But I admit that one has to (emotionally) cross a boundary of feeling comfortable with fundamental additions to a program like \TeX . But I've been around long enough to not feel guilty about it.

So in the end that means that for instance marks were extended, inserts got more options, glyphs and boxes have way more properties, (the result and handling of) paragraphs can be better controlled, page breaking got hooks (and might be extended), local boxes got redone, adjustments were extended, the math machinery has been completely opened up, hyphenation became more powerful, the font mechanism got more control and new scaling features, alignments got some extensions, we can do more with boxes, etc. But often I still first had to convince myself that it's okay to do so. After all, none of this had happened before and to my knowledge also has not been considered in ways that resulted in an implementation (but I might be wrong here). It helps that I can test out experiments in production versions of L \TeX and that users are quite willing to test.

2.5 Extending the macro language

In the previous section some mechanisms were mentioned, but before \TeX even ends up there macros and primitives come into play. The Lua \TeX engine already has some handy extras, like ways to prepend and append tokens and a limited so called 'local control' mechanism (think of nested main loops). There are some new look head and expansions related primitives and csname related tricks. There are a few more conditionals too. Details can be found in manual and articles.

In LuaMeta \TeX some more got added and some of these mechanism could be improved and the reason again is that I aim at readable code. Most programming languages for instance have conditionals with some kind of continuation (like `elseif`) and so I added that to \TeX too `\orelse`. Actually, there are even more new conditionals than in Lua \TeX . Yes, we don't really need these, especially because in LuaMeta \TeX we can now extend the primitive language via Lua, but I wanted to improve readability deep down in Con \TeX t. It also reduces the clutter when logging, although logging itself has been quite a bit overhauled. There is less need for intermediate (often not that natural) intermediate layers when we can do it properly in primitive \TeX lingua.

More fundamental was extending the way \TeX deals with macro arguments. Although the extensions to parsing them are using specifiers that make them upward compatible I admit that even I have to consult a list of possibilities every now and then but in the end they make things better (performance wise with less code). As a side effect the macro machinery could be optimized a bit (expansion as well as the save stacking).

There are a few more ways to store integers and dimensions (these fit in nicely), there are new into grouping, some primitives have more keywords and therefore scanners have been extended, the \TeX expression handlers have alternative variants.

Although this is a sensitive aspect of \TeX when it comes to compatibility, at some point I decided that it made no sense to not expose more details about nodes, input, and nesting states. The grouping and input related stacks had been optimized in the meantime so reporting in that area was already not compatible. Improving logging is an ongoing effort and I don't really loose sleep over it not being compatible, as long as it gets better. There is now also some tracing for marks, inserts, math and alignments.

2.6 Refactoring the code base

This is again an emotionally laden decision: what to touch and keep. For sure we keep the original comments but that doesn't make it literate. We started out with a C base that came from converted Pascal web.

The input machinery is a bit different due to the fact that Lua can (and often has to) kick in. In LuaMetaTeX it's even more different because even more goes via Lua. We cannot even run the engine without a basic set of callbacks assigned: if you don't like that, use LuaTeX. Does this violate the TeX concept? Not really, because system dependencies are explicitly mentioned as such in the source code. We have to adapt to the way an operating system sees files anyway (eight bit, utf8, utf16).

We still have many global variables (a practical Knuth thing I guess) but now they are grouped into structures so that we can more clearly see where they belong. This involved quite a bit of shuffling and editing but I got there. In LuaMetaTeX all constants (coded in macros) became enumerations, and all hard coded values too which was quite a bit of work too. Probably no one will notice or realize that, but starting from an existing code base is more work than starting from scratch, which is what I always did so far. When possible we use case statements. Most macros became (inline) functions. Complex functions got better variable names. All functions are in name spaces. This was (and is) a stepwise process that takes lots of time, especially because ConTeXt users expect a reasonable stable system and changes like that are sensitive for errors.

Talking of errors, the error and reporting system has been overhauled, so for instance we have now a dedicated string formatter. This all happened in several steps: normalization, consistency, abstraction, formatters, etc. Keep in mind that we not only have the original messages but also new ones. And we have TeX, Lua and MetaPost communicating with the user. Where in LuaTeX we have to conform more to the traditional engine, because that is what other macro packages rely on, In ConTeXt we have more freedom, so we can make it better and more detailed. Of course it could all be controlled by configurations but at some point I decided to kick out variables doing that because it made no sense to complicate the code base.

Memory management has been overhauled (more dynamic) as has dumping to the (more efficient) format file. With what is mentioned in the previous paragraphs we can safely say that in the meantime back porting to LuaTeX (which I had in mind) makes no sense any longer. There is occasionally some pressure to let LuaTeX do the same as other engines (new common features) and that doesn't always fit into the model. There is no need for LuaMetaTeX to follow up on that because often we already have plenty of possibilities. There is of course still work todo, for instance I still have to make some variable names in functions more verbose but that is not fundamental. I also have to go over the documentation in the code. I might make some interfaces more consistent anyway, so that also would demand adaptations. And of course the documentation in general always lags behind.

So far I only mentioned dealing with TeX, but keep in mind that in LuaMetaTeX we also have an upgraded MetaPost: only a Lua backend (we can produce pdf from that other output), no font code, a couple of extensions, more callbacks, io via Lua. Scanners make extending the language possible and injectors make for efficient piping back to MetaPost. Such extensions are also possible in TeX and the LuaMetaTeX scanning interfaces have been improved and extended too. We have extra callbacks (but some were dropped), more helpers (most noticeable in the node namespace), libraries that improve dealing with binary files, a reworked token library (which in turn lead to a reorganization of command codes in the TeX engine), a few more extensions if Lua file handling and string manipulations. We got decimal math, complex math, new compression libraries, better (Lua) memory management, a few optional library interfaces, etc. Fortunately that all didn't bloat the binary.

So, because in the meantime LuaMetaTeX is quite different from LuaTeX, we can consider the last one to be a prototype for the real deal.

2.7 Simplifying the build

This was one of the first things I did. It was a curious process of removing more and more of the original build (all kind of dependencies) which is not entirely trivial because of the way the LuaTeX build is set up. I

admit that I did try to stay within the regular source build concept but after a while I realized that this made no sense so we (Mojca was involved in that) made the move to cmake. Shortly after that I started using Visual Studio as editing environment (which saves time and is rather convenient) and native compilation under MS Windows became possible without any special measures (in fact, setting up the build for arm processors was more work).

A side effect is that right from the start we could provide binaries for various platforms via the compile farm on the ConT_EXt garden maintained by Mojca, who also does daily T_EX live builds there. On my machine I use the Windows Linux Subsystem for cross compilation but we can also do native builds. And, with my laptop being a robust 2013 old timer I force myself to make sure that LuaMetaT_EX keeps performing well.

2.8 Because it just makes sense

So, in the end LuaMetaT_EX is likely the engine most different from the Knuthian original but from the above one can conclude that this was a graduate process where I got more audacious over time. In the end the only thing that matters (and I believe that Don Knuth agrees with this) that you like writing the code, feel confident that the code is all right, explore the possibilities, try to improve the quality and understanding and that successive rewrites can reduce obscurity. And in my opinion we didn't lose the T_EX look and feel and still can operate well within the established boundaries of the T_EX ecosystem. The fact that most ConT_EXt users in the meantime use LuaMetaT_EX and the related LMTX variant is an indication that they are okay with it, and that is what matters most.

3 A new unit: dk

At the ConT_EXt 2021 meeting I mixed my T_EX talks with showing some of the (upcoming) LuaMetaT_EX source code. One evening we had a extension party where a new unit was implemented, the **dk**. This event was triggered by a remark Hraban [Ramm] made on the participants list in advance of the meeting, where he pointed to a Wikipedia article from which we quote:

“In issue 33, Mad published a partial table of the “Potrzebie System of Weights and Measures”, developed by 19-year-old Donald E. Knuth, later a famed computer scientist. According to Knuth, the basis of this new revolutionary system is the potrzebie, which equals the thickness of Mad issue 26, or 2.2633484517438173216473 mm [].”

So, as the result of that session, the source code now has this comment:

“We support the Knuthian Potrzebie, cf. en.wikipedia.org/wiki/Potrzebie, as the **dk** unit. It was added on 2021-09-22 exactly when we crossed the season during an evening session at the 15th ConT_EXt meeting in Bassenge (Boirs) Belgium. It took a few iterations to find the best numerator and denominator, but Taco Hoekwater, Harald Koenig and Mikael Sundqvist figured it out in this interactive session. The error messages have been adapted accordingly and the scanner in the Lua **tex** library also handles it. One **dk** is 6.43985pt. There is no need to make MetaPost aware of this unit because there it is just a numeric multiplier in a macro package.”

When compared to the already present units the **dk** nicely fills a gap:

unit	points	scaled	visual
sp	0.00002	1	
pt	1.0	65536	
bp	1.00374	65781	
dd	1.07	70124	
mm	2.84526	186467	█
dk	6.43985	422042	█
pc	12.0	786432	█
cc	12.8401	841489	█
cm	28.45274	1864679	█
in	72.26999	4736286	█

Deep down, the unit scanner uses a numerator and denominator in order to map the given value onto the internally used scaled points, so the relevant snippet of C is:

```
*num = 49838; // 152940;  
*denom = 7739; // 23749;  
return normal_unit_scanned;
```

The impact on performance of scanning an additional unit can be neglected because the scanning code is a bit different from the code in LuaT_EX and (probably the) other engines anyway.

Under consideration are a few extra units in the **relative_unit_scanned** category that we see in css: **vw** (relative to the `\hspace`), **vh** (relative to the `\vspace`), maybe a percentage (but of what) and **ch** (width of the current zero digit character). As usual with T_EXies, once it's there it will be (ab)used.

4 Anchoring

4.1 Introduction

It is valid to question what functionality should be in the engine and what can best be implemented using callbacks and postprocessing of lists (and boxes) relying for instance on attributes as signals. In LuaTeX we are rather strict in this and assume that the second method is used. In LuaMetaTeX we still promote this but at the same time some (lightweight) functionality has been added to the engine that helps implementing some features more efficiently. Reasons are that it can be handy to carry (fundamental) properties around that are bound to nodes and that we can set them using primitives, especially for glyphs and boxes. That way they become part of the formal functionality that one can argue should be present in a modern engine. Examples for glyph nodes are scales, offsets and hyphenation, detailed ligature and kerning control. For box nodes we have for instance offsets and orientation. Most of these are always taken into account by core mechanisms like breaking paragraphs into lines, where dimensions matter in which case it really makes sense for them to be part of the engine design.

Some properties are just passed on to for instance a font handler or the backend but still they belong to the core functionality. An example of the later is a (new) simple mechanism for anchoring boxes. This is not really a fundamental feature, because one can just move content around using a combination of kerning and boxing, either or not with offsets. But because we already have features like offsets to boxes it was not that much work to add anchoring as a more fundamental property. The frontend is agnostic to this feature because dimensions are kind of virtual here: the backend however carries the real burden. Because backends are written in Lua it might have a performance hit simply because at least we need to check if this feature is used. Normally that can be compensated when this feature *is* used because less work and shuffling around happens in the frontend. And when this feature is no longer experimental (and stays) we can gain some back by using it in existing scenarios. It sounds worse than it is because for orientations we already have to do some usage checking and we can share that check; in most situations nothing needs to be done anyway.

4.2 The low level approach

When we anchor, a box can be a source and/or a target. Both are represented by a number and can be assigned via a keyword. These numbers can be picked up by the backend. Here is an example:

```
\def\TestMe#1{%
  \setbox \scratchbox \ruledvbox
    source 123
    orientation #1
  \bgroup
    \hsize7cm
    \samplefile{zapf}
    \hbox to 0pt
      source 124 target 123
      xoffset 20pt yoffset -30pt
      {\darkred \bfc TEST1}%
    \hbox to 0pt
      source 125 target 124
      xoffset 10pt yoffset -20pt
      {\darkblue \bfc TEST2}%
}
```

```

\egroup
\box \scratchbox
}

```

This example also uses a few offsets. The ‘origin’ is at the left edge of the baseline. Now, we could have passed the source and target as attribute and intercepting an attribute in the backend can work pretty well. However, the code that deals with the final result of the typesetting and thereby flushes it to for instance a pdf file is, at least that is the setup we use in ConT_EXt, attribute agnostic. Mixing in attributes at that stage, except for user nodes and whatsits that are effectively plugins, is counter intuitive and all is already pretty complex so a clear separation of functionality makes a lot of sense. Of course the ConT_EXt approach is not the only one when it comes to generic engine functionality. Not that many fundamental (conceptual) extensions showed up over the last few decades so no one will bother if in LuaMetaT_EX we have new stuff that is only used by ConT_EXt. The example code shown here gives:

orientation 0 orientation 1 orientation 2 orientation 3

In order to avoid additional shifting around, which then might involve copying and injecting boxes as well as repackaging, two additional keys are available and these deal with the way boxes get anchored.

```

\ vbox
  source 123
  \ bgroup
    \ offinterlineskip
    \ blackrule[width=4cm,height=2cm,depth=0cm,color=darkred]\par
    \ blackrule[width=4cm,height=0cm,depth=1cm,color=darkblue]\par
    \ setbox\scratchboxtwo\ hbox
      anchors "0004 "0001
      % anchor "00040001
      target 123
      orientation 1
      {\blackrule[width=2cm,height=1cm,depth=0cm,color=darkgreen]}%
      \ hskip-2cm
      \ blackrule[width=2cm,height=0cm,depth=1cm,color=darkyellow]}%
      %
      \ smash{\box\scratchboxtwo}%
    \ egroup

```

The anchor is just a number but with the plural keyword we can scan it as two because that is a bit easier on usage. The two numbers four byte numbers control the source to target anchoring and there is plenty room for future extensions because not all bits are used.

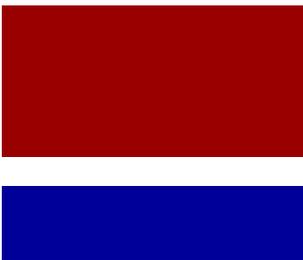
```

0x001 left origin
0x002 left height
0x003 left depth
0x004 right origin
0x005 right height
0x006 right depth
0x007 center origin
0x008 center height
0x009 center depth
0x00A halfway total
0x00B halfway height
0x00C halfway depth
0x00D halfway left
0x00E halfway right

```

The target and source are handled in a way that sort of naturally binds them which involves a little juggling with dimensions in the backend. There is some additional control over this but usage is not advertized here because it might change.

One can set these anchoring related properties with keywords but there are also primitive box manipulators: `\boxanchor`, `\boxanchors`, `\boxsource` and `\boxtarget` that take a box number and value.



There are some helpers at the Lua end but I haven't completely made up my mind about them. Normally that evolves with usage.

4.3 A first higher level interface

Exploring this here in more detail makes no sense because it is still experimental and also rather ConTeXt specific. As a teaser an interface that hooks into layers is shown:

```

\defineanchorboxoverlay[framed]

\def\DemoAnchor#1#2#3#4%
  {\setanchorbox
   [#1]%
   [target={#3},source={#4}]%
   \hbox{\backgroundline[#2]{\white\smallinfont\setstrut\strut target=#3
                                             source=#4}}}

\def\DemoAnchorX#1#2%
  {\DemoAnchor{#1}{darkred}   {#2}{left,top}%
   \DemoAnchor{#1}{darkblue}  {#2}{left,bottom}%
   \DemoAnchor{#1}{darkgreen} {#2}{right,bottom}%

```

```

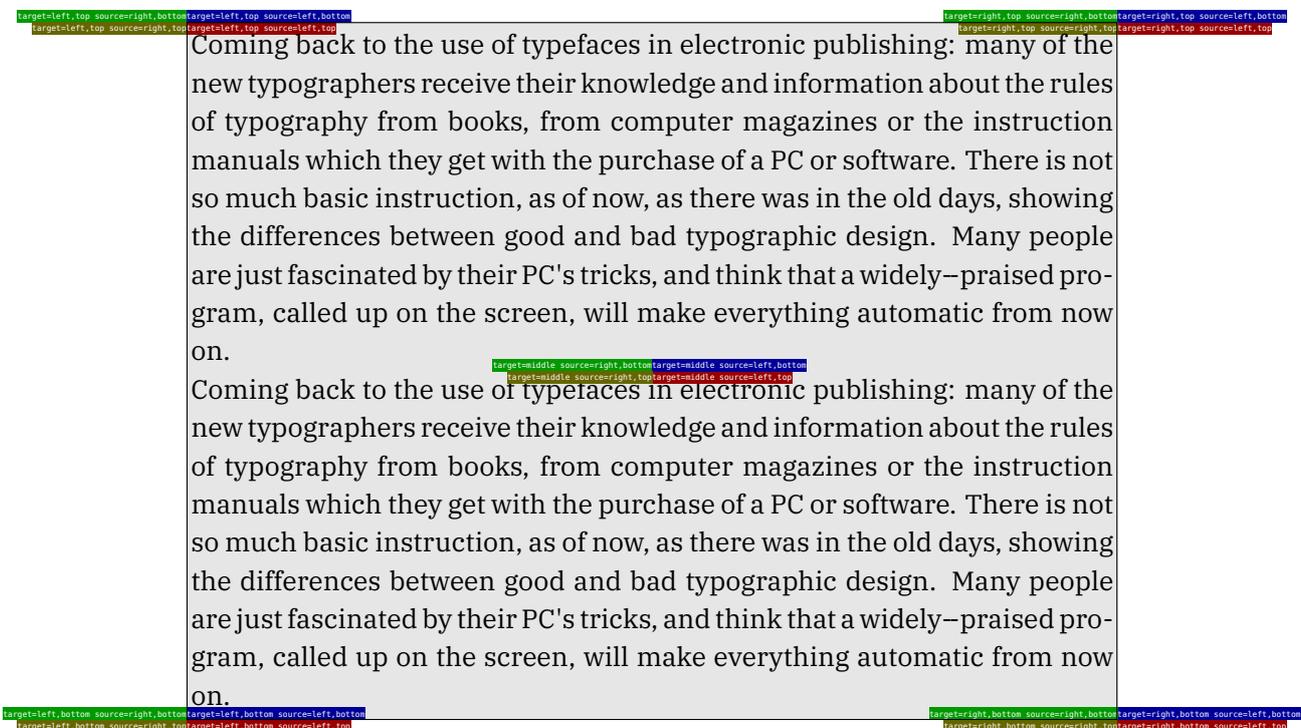
\DemoAnchor{#1}{darkyellow}{#2}{right,top}%
}%

\startsetups framed:demo
  \DemoAnchorX{framed:background}{left,top}%
  \DemoAnchorX{framed:background}{right,top}%
  \DemoAnchorX{framed:background}{left,bottom}%
  \DemoAnchorX{framed:background}{right,bottom}%
  \DemoAnchorX{framed:foreground}{middle}%
\stopsetups

\midaligned\bgroup
  \framed
    [align=normal,
     width=.7\textwidth,
     backgroundcolor=gray,
     background={color,framed:background,foreground,framed:foreground}]
  \bgroup
    \samplefile{zapf}\par
    \directsetup{framed:demo}%
    \samplefile{zapf}%
  \egroup
\egroup

```

Those familiar with ConTEXt will recognize the approach. This one basically is a more low level variant of layers and a high level variant of the primitives. Performance wise (in terms of memory usage and runtime) it sits in a sweet spot.



I played a bit with a mechanism that can store the embedded (to be anchored) content in a more independent way and it actually works okay. However, I'm not entirely sure if that solution is the best so for now it's commented. As usual it is also up to users to come up with demands.

5 A different approach to math spacing

Introduction

The \TeX engine is famous for its rendering of math and even after decades there is no real contender. And so there also is no real pressure to see if we can do better. However, when Mikael Sundqvist ran into a Swedish math rendering specification and we started discussing a possible support for that in Con \TeX t, it quickly became clear that the way \TeX does spacing is a bit less flexible than one wishes for. We already have much of what is needed in place but it also has to work well with how \TeX sees things:

1. Math is made from a sequence of atoms: a quantity with a nucleus, superscript subscript.⁴ Atoms are spaced by `\thinmuskip`, `\medmuskip` and `\thickmuskip` or nothing, and that is sort of hard coded.
2. Atoms are organized by class and there are seven (or eight, depending on how you look at it) of them visible: binary symbols, relations, etc. The invisible ones, composites like fractions and fenced material (we call them molecules) are at some point mapped onto the core set. Molecules like fences have a different class left and right of the fenced material.
3. In addition the engine itself has all kind of spacing related parameters and these kick in automatically and sometimes have side effects. The same is true for penalties.

The normal approach to spacing other than imposed by the engine is to use correction space, like `\,` and I think that quite some \TeX users think that this is how it is supposed to be. The standard way to enter math relates to scientific publishing and there the standards are often chiseled in stone so why should users tweak anyway. However, in Con \TeX t we tend to start from the users and not the publishers end so there we can decide to follow different routes. Users can always work around something they don't like but we focus on reliable input giving predictable output. Also, when reading on, it is good to realize that it is all about the user experience here: it should look nice (which then of course makes one become aware of issues elsewhere) and we don't care much about specific demands of publishers in the scientific field: the fact that they often re-key content doesn't go well with users paying attention themselves, let alone the fact that nowadays they can demand word processor formats.

The three mentioned steps are fine for the average case but sometimes make no sense. It was definitely the best approach given time and resources but when Lua \TeX went OpenType a lot of parameters were added and at that time we therefore added spacing by class pair. That not only decoupled the relation between the three (configurable) muskip parameters but also made it possible to use plenty of them. Now it must be said that for consistency having these three skips works great but given the tweaking expected from users consistency is not always what comes out.

This situation is very well comparable to the proclaimed qualities of the typesetting of text by \TeX . Yes, it can do a great job, and often does, but users can mess up quite well. I remember that when we did tests with hz the outcomes were pretty unimpressive. When you give an audience a set of sample renderings, where each sample is slightly different and each user gets a randomized subset, the sudden lack of being able to compare (and agree) with another \TeX ie makes for interesting conclusions. They look for the opposites of what is claimed to be perfect. So, two lines with hyphens rate low, even if not doing it would look worse. The same for a few short words in the last line of a paragraph. Excessive spacing is also seen as bad. So, when asked why some paragraphs looked okay noticing (excessive and troublesome) expansion was not seen as a problem; instead it were hyphens that got the attraction.

⁴ I suddenly realize why in the engine noads have a nucleus field: they are atoms . . . but what does that make super and subscripts.

The same is probably true for math: the input with lots of correction spaces or commands where characters would do can be horrible but it's just the way it is supposed to be. The therefore expected output can only be perfect, right, independent of how one actually messed up spacing. But personally I think that it is often spacing messed up by users that make a \TeX document recognizable. It compares to word processor results that one can sometimes identify by multiple consecutive spaces in the typeset text instead of using a glue model like \TeX . Reaching perfection is not always trivial, but fortunately we can also find plenty of nice looking documents done with \TeX .

The \TeX book has an excellent and intriguing chapter on the fine points of math and it definitely shows why Don Knuth wrote \TeX as a tool for his books. He pays a lot of attention to detail and that is also why it all works out so well. If you need to render from unseen sources (as happens in an xml workflow) coming from several authors and have time nor money to check everything, you're off worse. And I'm not even talking of input where invisible Unicode spacing characters are injected. It is the \TeX book(s) that has drawn me to this program and believe it or not, in the first project I was involved in that demanded typeset (quantum mechanics) math the ibm typewriter with changing bulbs ruled the scenery. In fact, our involvement was quickly cut off when we dared to show a chapter done in \TeX that looked better.

Apart from an occasional tweak, in Con \TeX t we never really used this opened up math atom pair spacing mechanism available in Lua \TeX extensively. So, when I was pondering how to proceed it stroke me that it would make sense to generalize this mechanism. It was already possible (via a mode parameter) to bypass the second step mentioned above, but we definitely needed more than the visible classes that the engine had. In Con \TeX t we already had more classes but those were meant for assigning characters and commands to specific math constructs (think of fences, fractions and radicals) so in the end they were not really classes. Considering this option was made easier by the fact that Mikael would do the testing and help configuring the defaults, which all will result in a new math user manual.

There are extensions introduced in Lua \TeX and later LuaMeta \TeX that are not discussed here. In this expose we concentrate on the features that were explored, extended and introduced while we worked on updating math support in LMTX.

An example

Before we go into details, let's give an example of unnoticed spacing effects. We use three simple formulas all using fractions:

```
\ruledhbox{$\frac{x^2}{a+1}$}
```

and:

```
\ruledhbox{$x + \frac{x^2}{a+1} = 10$}
```

as well as:

```
\ruledhbox{$\frac{1}{2}\frac{1}{2}x$}
```



If you look closely you see that the fraction has a little space at the left and right. Where does that come from? Because we normally don't put a tight frame around a fraction, we are not really aware of it. The spacing

between what are called ordinary, operator, binary, relation and other classes of atoms is explained in the \TeX book (or “ \TeX by Topic” if you want a summary) and basically we have a class by class matrix that is built into \TeX . The engine looks at successive items and spacing depends on their (perceived) class. Because the number of classes is limited, and because the spacing pairs are hard coded, the engine cheats a little. Depending on what came before or comes next the class of an atom is adapted to suit the spacing matrix. One can say that a “reading mathematician” is built in. And most of the decisions are okay. If needed one can always wrap something in e.g. `\mathrel` but of course that also can interfere with grouping. All this is true for \TeX , pdf \TeX , X \TeX and Lua \TeX , but a bit different in LuaMeta \TeX as we will see.

The little spacing on both edges of the fraction is a side effect of the way they are built internally: fractions are actually a generalized form of “stuff put on top of other stuff” and they can have left and/or right delimiters: this is driven by primitives that have names like `\atop` and `\atopwithdelims`. The way the components are placed is (especially in the case of OpenType) driven by lots of parameters and I will leave that out of the discussion.

When there are no delimiters, a so called `\nulldelimiterspace` will be injected. That parameter is set to 1.2 points and I have to admit that in Con \TeX t I never considered letting that one adapt to the body font size, which means that, as we default to a 12 point body font, the value there should have been 1.44 points: mea culpa. When we set this parameter to zero point, we get this:



As intermezzo and moment of contemplation I show some examples of fractions mixed into text. When we have the delimiter space set we get this:

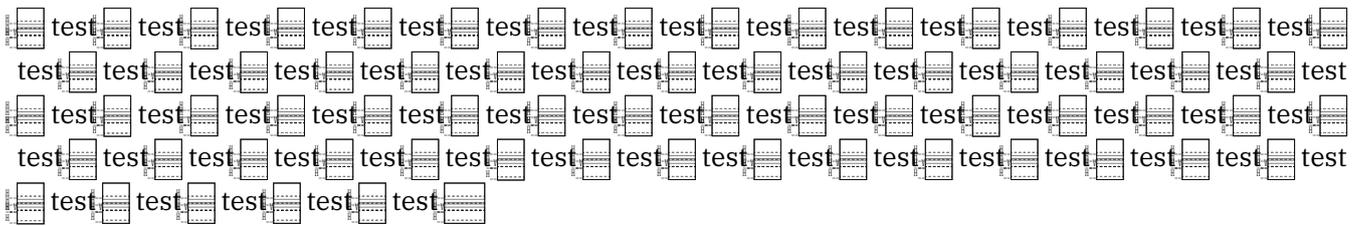
```
test test
test test test test test test test test test test test test test test test test test test
  test test test test test test test test test test test test test test test test test
test test test test test test test test test test test test test test test test test test
  test test test test test test test test test test test test test test test test test
test test test test test test test test test test test test test test test test test test
  test test test test test
```

While with zero it looks like this, quite a different outcome:

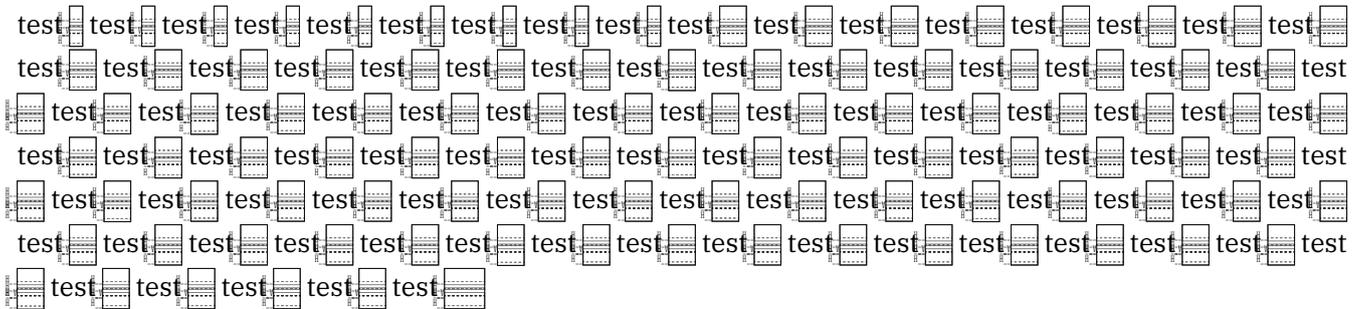
```
test test
test test test test test test test test test test test test test test test test test test
  test test test test test test test test test test test test test test test test test
test test test test test test test test test test test test test test test test test test
  test test test test test test test test test test test test test test test test test
test test test test test test test test test test test test test test test test test test
  test test test test test
```

A little tracing shows it more clearly:

```
test ☐ test ☐
test ☐ test ☐ test ☐ test ☐ test ☐ test ☐ test ☐ test ☐ test ☐ test ☐ test ☐ test ☐ test ☐ test ☐
```

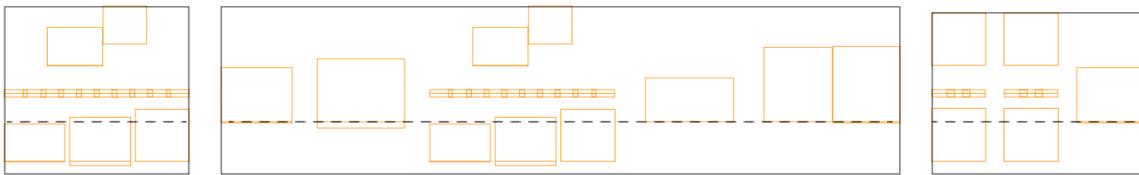


You can zoom in and see where it interferes with margin alignment.

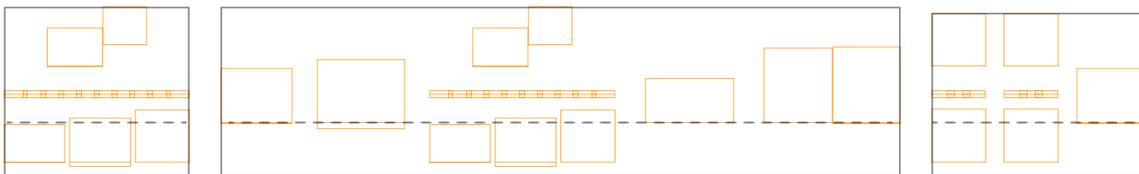


So, if you ever meet a user who claims perfection and superiority of typesetting, check out her/his work which might have inline fractions done the spacy way. It might make other visually typesetting claims less trustworthy. And yes, one can wonder if margin kerning could help here but as this content is wrapped in boxes it is unlikely to work out well (and not worth the effort).

In order to get a better picture of the spacing, two more renderings are shown. This time we show the bounding boxes of the characters too (you might need to zoom in to see it):

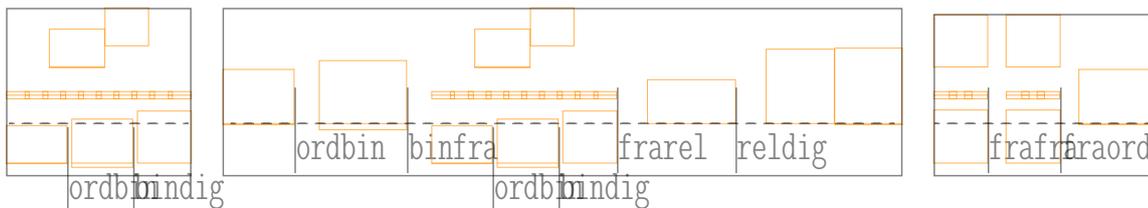


Again we also show the zero case

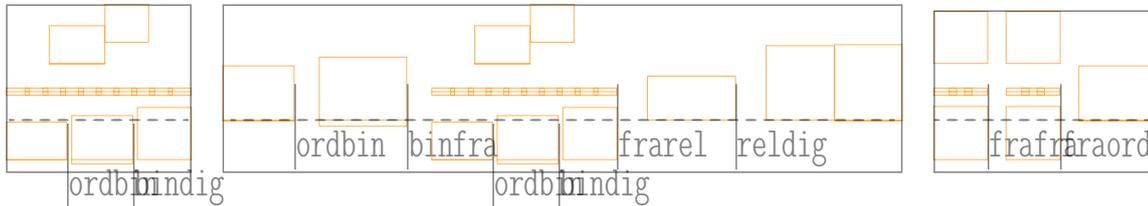


This makes clear why there actually is this extra space around a fraction: regular operators have side bearings and thereby have some added space. And when we put a fraction in front of a symbol we need that little extra space. Of course a proper class pair spacing value could do the job but there is no fraction class. The engine cheats by changing the class depending on what follows or came before and this is why on the average it looks okay. However, these examples demonstrate that there are some assumptions with regard to for instance fonts and this is one of the reasons why the more or less official expected OpenType behavior as dictated by the Cambria font doesn't always work out well for fonts that evolved from the ones used in the T_EX community. Also imagine how this interferes with the fact that traditional T_EX fonts and the machinery do magic with cheating about width combined with italic correction (all plausible and quite clever but somewhat tricky with respect to OpenType).

Because here we discuss the way LuaMetaT_EX and ConT_EXt deal with this, the following examples show a probably unexpected outcome. Again first the non-zero case:



And here the zero case:



I will not go into details about the way fractions are supported in the engine because some extensions are already around for quite a while. The main observation here is that in LuaMetaTeX we have alternative primitives that assume forward scanning, as if the numerator and denominator are arguments. The engine also supports skewed (vulgar) fractions natively where numerator and denominator are raised and lowered relative to the (often) slash. Many aspects of the rendering can be tuned in the so called font goodie files, which is also the place where we define the additional font parameters.

Atom spacing

If you are familiar with traditional TeX you know that there is some built in `ordbin` spacing. But there is no such pair for a fraction and a relation, simply because there is no fraction class. However, in LuaMetaTeX there is one, and we'd better set it up if we zero the margins of a fraction.

It is worth noticing that fractions are sort of special anyway. The official syntax is `n \over m` and numerator and denominator can be sub formulas. This is the one case where the parser sort of has to look back, which is tricky because the machinery is a forward looking one. Therefore, in order to get the expected styling (or avoid unexpected side effects) one will normally wrap all in braces as in: `{ {n} \over {m} }` which of course kind defeats the simple syntax which probably is supported for `1\over2` kind of usage, so a next challenge is to make `1/2` come out right. All this means that in practice we have wrappers like `\frac` which accidentally in LuaMetaTeX can be defined using forward looking primitives with plenty extra properties driven by keywords. It also means that fractions as expected by the engine due to wrapping actually can be a different kind of atom, which can have puzzling side effects with respect to spacing (because the remapping happens unseen).

Interesting is that adapting LuaMetaTeX to a more extensive model was quite doable, also because the code base had already been made more configurable. Of course it involved quite a bit of tedious editing and throwing out already nice and clean code that had taken some effort, but that's the way it is. Of course more classes also means that some storage properties had to be adapted within the available space but by sacrificing families that was possible. With 64 potential classes we now are back to 64 families compared to 7 classes and 256 families in LuaTeX and 7 classes and 16 families in traditional TeX.

Also interesting is that the new implementation is actually somewhat simpler and therefore the binary is a tad smaller too. But does all that mean that there were no pitfalls? Sure there were! It is worth noticing that doing all this reminded me of the early days of LuaTeX development, where Taco and I exchanged binaries and TeX code in a more or less constant way using Skype. For LuaMetaTeX we used good old mail for files and Mojca's build farm for binaries and Mikael and I spent many months exchanging information and testing out alternatives on a daily basis: it is in my opinion the only way to do this and it's fun too. It has been a lot

of work but once we got going there was nothing that could stop us. A side effect was that there were no updates during this period, which was something users noticed.

In the spacing matrix there is `inner` and internally there's also some care to be taken of `vcenter`. The `inner` class is actually shared with the `variable` class which is not so much a real class but more a signal to the engine that when an alphabetic or numeric character is included it has to come from a specific family: upright family zero or math italic family one in traditional speak. But, what if we don't have that setup? Well, then one has to make sure that this special class number is not associated (which is no big deal). It does mean that when we extend the repertoire of classes we cannot use slot seven. Always keep in mind that classes (and thereby signals) get assigned to characters (some defaults by the engine, others by the macro package). It is why in ConT_EXt we use abstract class numbers, just in case the engine gets adapted.

We also cannot use slot eight because that one is a signal too: for a possible active math character, a feature somewhat complicated by the fact that it should not interfere with passing around such active characters in arguments. In math mode where we have lots of macros passing around content, this special class works around these side effects. We don't need this feature in ConT_EXt because contrary to other macro packages we don't handle primes, pseudo superscripts potentially followed by other super and subscripts by making the ' an active character and thereby a macro in math mode. This trickery again closely relates to preferable input, font properties, and limitations of memory and such at the time T_EX showed up (much has to fit into 8, 16 or 32 bits, so there is not much room for e.g. more than 8 classes). Since we started with MkIV the way math is dealt with is a bit different than normally done in T_EX anyway.

Atom rules

We can now control the spacing between every atom but unfortunately that is not good enough. Therefore, we arrive at yet another feature built into the engine: turning classes into other classes depending on neighbors. And this is precisely why we have certain classes. Let's quote “T_EX by Topic”: *The cases `*` (in the atom spacing matrix) cannot occur, because a `bin` object is converted to `ord` if it is the first in the list, preceded by `bin`, `op`, `open`, `punct`, `rel`, or followed by `close`, `punct`, or `rel`; also, a `rel` is converted to `ord` when it is followed by `close` or `punct`.*

We can of course keep these hard coded heuristics but can as well make that bit of code configurable, which we did. Below is demonstrated how one can set up the defaults at the T_EX end. We use symbolic names for the classes.

```

\setmathatomrule \mathbegincode      \mathbinarycode      % old
  \allmathstyles  \mathordinarycode    \mathordinarycode    % new

\setmathatomrule \mathbinarycode      \mathbinarycode
  \allmathstyles  \mathbinarycode    \mathordinarycode
\setmathatomrule \mathoperatorcode    \mathbinarycode
  \allmathstyles  \mathoperatorcode  \mathordinarycode
\setmathatomrule \mathopencode        \mathbinarycode
  \allmathstyles  \mathopencode      \mathordinarycode
\setmathatomrule \mathpunctuationcode \mathbinarycode
  \allmathstyles  \mathpunctuationcode \mathordinarycode
\setmathatomrule \mathrelationcode    \mathbinarycode
  \allmathstyles  \mathrelationcode  \mathordinarycode

\setmathatomrule \mathbinarycode      \mathclosecode

```

```

\allmathstyles \mathordinarycode \mathclosecode
\setmathatomrule \mathbinarycode \mathpunctuationcode
\allmathstyles \mathordinarycode \mathpunctuationcode
\setmathatomrule \mathbinarycode \mathrelationcode
\allmathstyles \mathordinarycode \mathrelationcode

\setmathatomrule \mathrelationcode \mathclosecode
\allmathstyles \mathordinarycode \mathclosecode
\setmathatomrule \mathrelationcode \mathpunctuationcode
\allmathstyles \mathordinarycode \mathpunctuationcode

```

Watch the special class with `\mathbegincode`. This is actually class 62 so you don't need much fantasy to imagine that class 63 is `\mathendcode`, but that one is not yet used. In a similar fashion we can initialize the spacing itself.⁵

```

\setmathspacing \mathordcode \mathopcode \allmathstyles \thinmuskip
\setmathspacing \mathordcode \mathbincode \allsplitstyles \medmuskip
\setmathspacing \mathordcode \mathrelcode \allsplitstyles \thickmuskip
\setmathspacing \mathordcode \mathinnercode \allsplitstyles \thinmuskip

\setmathspacing \mathopcode \mathordcode \allmathstyles \thinmuskip
\setmathspacing \mathopcode \mathopcode \allmathstyles \thinmuskip
\setmathspacing \mathopcode \mathrelcode \allsplitstyles \thickmuskip
\setmathspacing \mathopcode \mathinnercode \allsplitstyles \thinmuskip

\setmathspacing \mathbincode \mathordcode \allsplitstyles \medmuskip
\setmathspacing \mathbincode \mathopcode \allsplitstyles \medmuskip
\setmathspacing \mathbincode \mathopencode \allsplitstyles \medmuskip
\setmathspacing \mathbincode \mathinnercode \allsplitstyles \medmuskip

\setmathspacing \mathrelcode \mathordcode \allsplitstyles \thickmuskip
\setmathspacing \mathrelcode \mathopcode \allsplitstyles \thickmuskip
\setmathspacing \mathrelcode \mathopencode \allsplitstyles \thickmuskip
\setmathspacing \mathrelcode \mathinnercode \allsplitstyles \thickmuskip

\setmathspacing \mathclosecode \mathopcode \allmathstyles \thinmuskip
\setmathspacing \mathclosecode \mathbincode \allsplitstyles \medmuskip
\setmathspacing \mathclosecode \mathrelcode \allsplitstyles \thickmuskip
\setmathspacing \mathclosecode \mathinnercode \allsplitstyles \thinmuskip

\setmathspacing \mathpunctcode \mathordcode \allsplitstyles \thinmuskip
\setmathspacing \mathpunctcode \mathopcode \allsplitstyles \thinmuskip
\setmathspacing \mathpunctcode \mathrelcode \allsplitstyles \thinmuskip
\setmathspacing \mathpunctcode \mathopencode \allsplitstyles \thinmuskip
\setmathspacing \mathpunctcode \mathclosecode \allsplitstyles \thinmuskip
\setmathspacing \mathpunctcode \mathpunctcode \allsplitstyles \thinmuskip
\setmathspacing \mathpunctcode \mathinnercode \allsplitstyles \thinmuskip

\setmathspacing \mathinnercode \mathordcode \allsplitstyles \thinmuskip

```

⁵ Constant, engine specific, numbers like these are available in tables at the Lua end so we can change them and users can check that.

```

\setmathspacing \mathinnercode \mathopcode      \allmathstyles \thinmuskip
\setmathspacing \mathinnercode \mathbincode     \allsplitstyles \medmuskip
\setmathspacing \mathinnercode \mathrelcode     \allsplitstyles \thickmuskip
\setmathspacing \mathinnercode \mathopencode    \allsplitstyles \thinmuskip
\setmathspacing \mathinnercode \mathpunctcode   \allsplitstyles \thinmuskip
\setmathspacing \mathinnercode \mathinnercode   \allsplitstyles \thinmuskip

```

And because we have a few more atom classes this also needs to happen:

```

\letmathspacing \mathactivecode  \mathordinarycode
\letmathspacing \mathvariablecode \mathordinarycode
\letmathspacing \mathovercode     \mathordinarycode
\letmathspacing \mathundercode    \mathordinarycode
\letmathspacing \mathfractioncode \mathordinarycode
\letmathspacing \mathradicalcode  \mathordinarycode
\letmathspacing \mathmiddlecode   \mathopencode
\letmathspacing \mathaccentcode   \mathordinarycode

```

```

\letmathatomrule \mathactivecode  \mathordinarycode
\letmathatomrule \mathvariablecode \mathordinarycode
\letmathatomrule \mathovercode     \mathordinarycode
\letmathatomrule \mathundercode    \mathordinarycode
\letmathatomrule \mathfractioncode \mathordinarycode
\letmathatomrule \mathradicalcode  \mathordinarycode
\letmathatomrule \mathmiddlecode   \mathopencode
\letmathatomrule \mathaccentcode   \mathordinarycode

```

With `\resetmathspacing` we get an all-zero state but that might become more refined in the future. What is not clear from the above is that there is also an inheritance mechanism. The three special muskip registers are actually shortcuts so that changing the register value is reflected in the spacing. When a regular muskip value is (verbose or as register) that value is sort of frozen. However, the `\inherited` prefix will turn references to registers and constants into a delayed value: as with the predefined we now have a more dynamic behavior which means that we can for instance use reserved muskip registers as we can use the predefined. A bonus is that one can also use regular glue or dimensions, just in case one wants the same spacing in all styles (a muskip adapts to the size).

When you look at all of the above you might wonder how users are supposed to deal with math spacing. The answer is that often they can just assume that \TeX does the right thing. If something somehow doesn't feel right, looking at solutions by others will probably lead a new user to just copy a trick, like injecting a `\thinmuskip`. But it can be that atoms depend on the already applied (or not) spacing, which in turn depends on values in the atom spacing matrix that probably only a few users have seen. So, in the end it all boils down to trust in the engine and one's eyesight combined with hopefully some consistency in adding space directives and often with \TeX it is consistency that makes documents look right. In $\text{Con}\TeX$ t we have many more classes even if only a few characters fit in, like differential, exponential and imaginary.

Fractions again

We now return to the fraction molecule. With the mechanisms at our disposal we can change the fixed margins to more adaptive ones:

```

\inherited\setmathspacing \mathbinarycode  \mathfractioncode

```

```

\allmathstyles \thickermskip
\inherited\setmathspacing \mathfractioncode \mathbinarycode
\allmathstyles \thickermskip
\nulldelimiterspace\zeropoint
$x + \frac{1}{x+2} + x$

```

Here `\thickermskip` is defined as `7mu plus 5mu` where the stretch is the same as a `\thickmskip` and the width `2mu` more. We start out with three variants, where the last two have `\nulldelimiterspace` set to `0pt` and the first one uses the `1.2pt`.

When we now apply the new settings to the last one, and overlay them we get the following output: the first and last case are rather similar which is why this effort was started in the first place.

Of course these changes are not upward compatible but as they are tiny they are not that likely to change the number of lines in a paragraph. In display mode changes in horizontal dimensions also have little effect.

Penalties

An inline formula can be broken across lines, and for sure there are places where you don't want to break or prefer to break. In \TeX line breaks can be influenced by using penalties. At the outer level of an inline math formula, we can have a specific penalty before and after a binary and/or relation. The defaults are such that there are no penalties set, but most macro packages set the so called `\relpenalty` and `\binoppenalty` (the `op` in this name does not relate to the operator class) so a value between zero and 1000. In $\text{Lua}\TeX$ we also have `\pre` variants of these, so we have four penalties that can be set, but that is not enough in our new approach.

These penalties are class bound and don't relate to styles, like atom spacing does. That means that while atom spacing involves potential values, an amount that we can manage by using the discussed inheritance. The inheritance takes less values because which store 4 style values per class in one number. For penalties we only need to keep in mind, plus a range of inheritance numbers. Therefore it was

decided to also generalize penalties so that each class can have them. The magic commands are shown with some useless examples:

```
\letmathparent \mathdigitcode
  \mathbincode % pre penalty
  \mathbincode % post penalty
  \mathdigitcode % options
  \mathdigitcode % reserved
```

By default the penalties are on their own, like:

```
\letmathparent \mathdigitcode
  \mathdigitcode % pre penalty
  \mathdigitcode % post penalty
  \mathdigitcode % options
  \mathdigitcode % reserved
```

The options and reserved parent mapping are not (yet) discussed here. Unless values are assigned they are ignored.

```
\setmathprepenalty \mathordcode 100
\setmathpostpenalty \mathordcode 600
\setmathprepenalty \mathbincode 200
\setmathpostpenalty \mathbincode 700
\setmathprepenalty \mathrelcode 300
\setmathpostpenalty \mathrelcode 800
```

As with spacing, when there is no known value, the parent will be consulted. An unset penalty has a value of 10000.

After discussing the implications of inline math crossing lines, Mikael and I decided there can be two solutions. Both can of course be implemented in Lua, but on the other hand, they make good extensions, also because it sort of standardized it. The first advanced control feature tweaks penalties:

```
\mathforwardpenalties 2 200 100
\mathbackwardpenalties 2 100 50
```

This will add 200 and 100 to the first two math related penalties, and 100 and 50 to the last two (watch out: the 100 will be assigned to the last one found, the 50 to the one before it). As with all things penalty and line break related, you need to have some awareness of how non-linear the badness calculation is as well of the fact that the tolerance and stretch related parameters play a role here.

The second tweak is setting `\maththreshold` to some value. When set to for instance `40pt`, formulas that take less space than this will be wrapped in a `\hbox` and thereby will never break across a page.⁶ Actually that second tweak has a variant so we have three tweaks! Say that we have this sample formula wrapped in some bogus text and repeat that snippet a lot of times:

```
x xx xxx xxxx $1 + x$ x xx xxx xxxx
```

Now look at the example on the next page. You will notice that the red and blue text have different line breaks. This is because we have given the threshold some stretch and shrink. The red text has a zero threshold so it doesn't do any magic at all, while the second has this setup:

⁶ A future version might inject severe penalties instead, time will learn.

```
\setupmathematics[threshold=medium]
```

That setting set the threshold to `4em plus 0.75em minus 0.50em` and when the formula size exceeds the four quads the line break code will use the real formula width but with the given stretch and shrink. Eventually the calculated size will be used to repackage the formula. In the future we will also provide a way to define slack more relative to the size and/or number of atoms.

Another way to influence line breaks is to use the two inline math related penalties that have been added at Mikael's suggestion:

```
\setupalign [verytolerant]
{\dorecurse{25}{test $\darkred #1^{#1} + x_{#1}^{#1}$ test }\blank}
{\preinlinepenalty 500 \postinlinepenalty -500
 \dorecurse{25}{test $\darkgreen #1^{#1} + x_{#1}^{#1}$ test }\blank}
{\postinlinepenalty 500 \preinlinepenalty -500
 \dorecurse{25}{test $\darkblue #1^{#1} + x_{#1}^{#1}$ test }\blank}
```

To get an example that shows the effect takes a bit of trial and error because \TeX does a very good job in line breaking. This is why we've set the tolerance and also use negative penalties.

In addition to the `\mathsurround` (kern) and `\mathsurroundskip` (glue) parameters this is a property of the nodes that mark the beginning and end of an inline math formula.

```
test      test test      test test      test test      test test      test test      test test
      test test      test test      test test      test test      test test
test test      test test      test test      test test      test test      test test      test
test      test test      test test      test test      test test      test test      test test
      test test      test test      test
test      test test      test test      test test      test test      test test      test test
test test      test test      test test      test test      test test      test test      test test
      test test      test test      test test      test test      test test
test test      test test      test test      test test      test test      test test      test
test      test test      test test      test test      test test      test test      test test
      test test      test test      test test      test test      test test      test test
test test      test test      test test      test test      test test      test test      test test
      test test      test
```

Flattening

The traditional engine has some code for flattening math constructs that in the end are just one character. So in the end, `\tilde{u}` and `\tilde{uu}` become different objects even if both are in fact accents. In fact, when an accent is constructed there is a special code path for single characters so that script placement adapts to the shape of that character.

However because of interaction with primes, which themselves are sort of superscripts and due to the somewhat weird way fonts provide them when it comes to positioning and sizes, in Con \TeX t we already are fooling

```

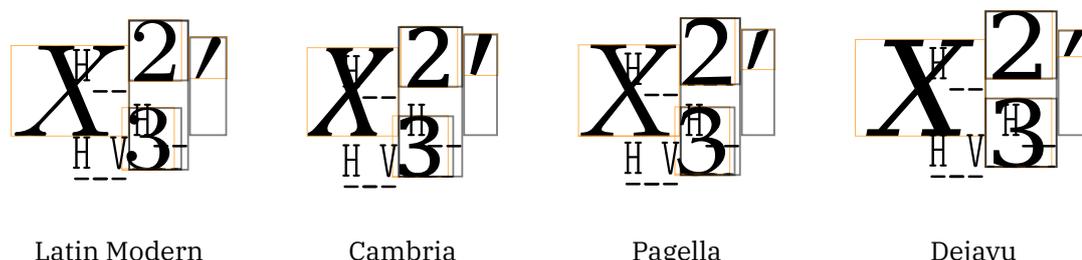
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
XXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX X XX XXX XXXX      X XX XXX XXXX
XXXX X XX XXX XXXX      X XX XXX XXXX

```

around a bit with these characters. For understandable reasons of memory usage, complexity and eightbit-ness primes are not a native \TeX thing but more something that is handled at the macro level (although not in MkIV and LMTX).

In the end it was script placements on (widely) accented math characters that made us introduce a dedicated `\Umathprime` primitive that adds a prime to a math atom. It permits an uninterrupted treatment of scripts while in the final assembly of the molecule the prime, superscript, subscript and maybe even prescripts that prime gets squeezed in. Because the concept of primes is missing in OpenType math an additional font parameter `PrimeTopRaisePercent` has been introduced as well as an `\Umathprimeraise` primitive. In retrospect I should have done that earlier but one tends to stick to the original as much as possible. However, at some point Mikael and I reached a state where we decided that proper (clean) engine extensions make way more sense than struggling with border cases and explaining users why things are so complicated.

The input `$ X \Uprimescript{'} ^2 _3 $` gives this:



With `\tracingmath = 1` this nicely traces as:

```

> \inlinemath=
\noad[ord][...]
.\nucleus
..\mathchar[ord] family "0, character "58
.\superscript
..\mathchar[dig] family "0, character "32
.\subscript
..\mathchar[dig] family "0, character "32
.\primescript
..\mathchar[ord] family "0, character "27

```

Of course this feature can also be used for other prime like ornaments and who knows how it will evolve over time.

You can influence the positioning with `\Umathprimesupshift` which adds some kern between a prime and superscript. The `\Umathextraprimeshift` moves a prime up. The `\Umathprimeraise` is a font parameter that defaults to 25 which means a raise of 25% of the height. These are all (still) experimental parameters.

Fences

Fences can be good for headaches. Because the math that I (or actually my colleague) deal with is mostly school math encoded in presentation MathML (sort or predictable) or some form of sequential ascii based input (often rather messy and therefore unpredictable due to ambiguity) fences are a pain. A \TeX ie can make sure that left and right fences are matched. A \TeX ie also knows when something is an inline parenthesis or when a more high level structure is needed, for instance when parentheses have to scale with what they wrap. In that case the `\left` and `\right` mechanism is used. In arbitrary input missing one of those is fatal. Therefore, handling of fences in Con \TeX t is one of the more complex sub mechanisms: we not only need to scale when needed, but also catch asymmetrical usage.

A side effect of the encapsulating fencing construct is that it wraps the content in a so called inner (as in `\mathinner`) which means that we get a box, and it is a well known property of boxes that they don't break across lines. With respect to fences, a way out is to not really fence content, but do something like this:

```
\left(\strut\right. x + 1 \left.\strut\right)
```

and hope for the best. Both pairs are coupled in the sense that their sizes will match and the strut is what determines the size. So, as long as there is a proper match of struts all is well, but it is definitely a decent hack. The drawback is in the size of the strut: if a formula needs a higher one, larger struts have to be used. This is why in plain \TeX we have these commands:

```
\def\bigl {\mathopen \big } \def\bigm {\mathrel\big } \def\bigR {\mathclose\big }
\def\Bigl {\mathopen \Big } \def\Bigm {\mathrel\Big } \def\BigR {\mathclose\Big }
\def\biggl {\mathopen \bigg} \def\biggm {\mathrel\bigg} \def\biggr {\mathclose\bigg}
\def\Biggl {\mathopen \Bigg} \def\Biggm {\mathrel\Bigg} \def\Biggr {\mathclose\Bigg}
```

```
\def\big #1{\hbox{$\left#1\ vbox to 8.5pt}\right.\nomathspacing$}}
\def\Big #1{\hbox{$\left#1\ vbox to 11.5pt}\right.\nomathspacing$}}
\def\bigg#1{\hbox{$\left#1\ vbox to 14.5pt}\right.\nomathspacing$}}
\def\Bigg#1{\hbox{$\left#1\ vbox to 17.5pt}\right.\nomathspacing$}}
```

```
\def\nomathspacing{\null\delimiterspace0pt\mathsurround0pt} % renamed
```

The middle is kind of interesting because it has relation properties, while the `\middle` introduced in \TeX got open properties, but we leave that aside.

In Con \TeX t we have plenty of alternatives, including these commands, but they are defined differently. For instance they adapt to the font size. The hard coded point sizes in the plain \TeX code relates to the font and steps available in there (either by next larger or by extensible). The values thereby need to be adapted to the chosen body font as well as the body font size. In MkIV and even better in L \TeX we can actually consult the font and get more specific sizes.

But, this section is not about how to get these fixed sizes. Actually, the need to choose explicitly is not what we want, especially because \TeX can size delimiters so well. So, take this code snippet:

```
$ x = \left( \dorecurse{40}{\frac{x}{x+#1} +} x \right) $
```

When we typeset this inline, as in

we get nicely scaled fences but in a way that permits line breaks. The reason is that the engine has been extended with a `fenced` class so that we can recognize later on, when \TeX comes to injecting spaces and penalties, that we need to unpack the construct. It is another beneficial side effect of the generalization.

The Plain T_EX code can be used to illustrate some of what we discussed before about fractions. In the next code we use excessive delimiter spacing:

```
\def\Bigg#1{% watch the wrapping in a box
  {%
    \hbox {%
      $\normalleft#1\vbox to 17.5pt{}\normalright.\nomathspacing$%
    }%
  }%
}
```

```
\nulldelimiterspace0pt
\def\nomathspacing{\nulldelimiterspace0pt\mathsurround0pt}
```

```
$$\Bigg( 1 + x\Bigg) \quad \Bigg( \frac{1}{x}\Bigg)$$\par
```

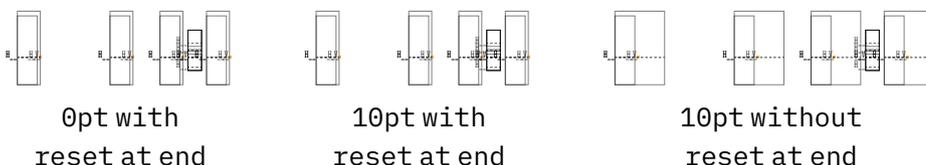
```
\nulldelimiterspace10pt
\def\nomathspacing{\nulldelimiterspace0pt\mathsurround0pt}
```

```
$$\Bigg( 1 + x\Bigg) \quad \Bigg( \frac{1}{x}\Bigg)$$\par
```

```
\nulldelimiterspace10pt
\def\nomathspacing{\mathsurround0pt}
```

```
$$\Bigg( 1 + x\Bigg) \quad \Bigg( \frac{1}{x}\Bigg)$$\par
```

This renders as follows. We explicitly set `\nulldelimiterspace` to values because in ConT_EXt it is now zero by default.



Radicals

In traditional T_EX a radical with degree is defined as macro. That macro does some measurements and typesets the result in four sizes for a choice. The macro typesets the degree in a box that contains the degree as formula. There is a less guesswork going on than with respect to how the radical symbol is shaped but as we're talking plain T_EX here it works out okay because the default font is well known.

Radicals are a nice example of a two dimensional ‘extender’ but only the vertical dimension uses the extension mechanism, which itself operates either horizontally or vertically, although in principle it could go both ways. The horizontal extension is a rule and the fact that the shape is below the baseline (as are other large symbols) will make the rule connect well: the radical shape sticks out a little, so one can think of the height reflecting the rule height.⁷ In OpenType fonts there is a parameter and in LuaT_EX we use the default rule thickness for traditional fonts, which is correct for Latin Modern. There are more places in the fonts

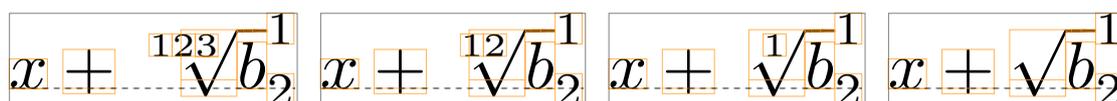
⁷ When you zoom in you will notice that this is not always optimal because of the way the slope touched the rule.

where the design relates to this thickness, for instance fraction rules are supposed to match the minus, but this is a bit erratic if you compare fonts. This is one of the corrections we apply in the goodie files.

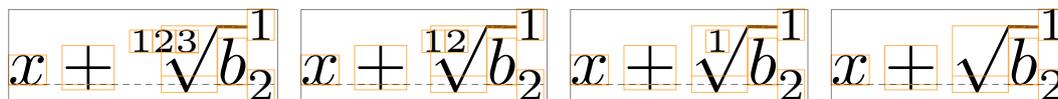
In OpenType the specification of the radical also includes spacing properties of the degree and that is why we have a primitive in LuaTeX that also handles the degree. It is what we used in ConTeXt MkIV. But . . . we actually end up with a situation that compares to the already discussed fraction: there is space added before a radical when there is a degree. However, because we now have a radical atom class, we can avoid using that one and use the new pairwise spacing. Some fuzzy spacing logic in the engine could therefore be removed and we assume that `\Umathradicaldegreebefore` is zero. For the record: the `\Umathradicaldegreeafter` sort of tells how much space there is above the low part of the root, which means that we can compensate for multi-digit degrees.

Zeroing a parameter is something that relates to a font which means that it has to happen for each math font which in turn can mean a family-style combination. In order to avoid that complication (or better: to avoid tracing clutter) we have a way to disable a parameter:

```
\ruledhbox{$x + \sqrt[123]{b}^1_2$}
\ruledhbox{$x + \sqrt[12]{b}^1_2$}
\ruledhbox{$x + \sqrt[1]{b}^1_2$}
\ruledhbox{$x + \sqrt{b}^1_2$}
```



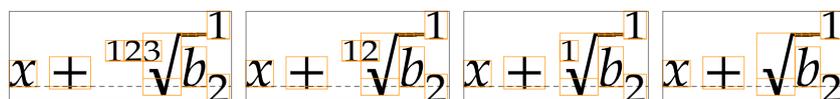
`\setmathignore\Umathradicaldegreebefore 0`



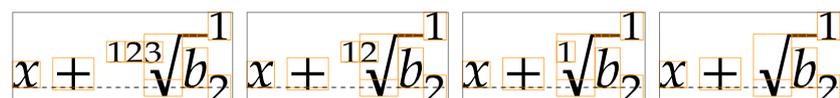
`\setmathignore\Umathradicaldegreebefore 1`

Latin Modern

One problem with these spacing parameters is that they are inconsistent across fonts. The Latin Modern has a rather large space before the degree, while Cambria and Pagella have little. That means that when you prototype a mechanism the chosen solution can look great but not so much when at some point you use another font.



`\setmathignore\Umathradicaldegreebefore 0`



`\setmathignore\Umathradicaldegreebefore 1`

Cambria

More fences

One of the reasons why the MkII and MkIV fence related mechanism is somewhat complex is that we want a clean solution for filtering fences like parenthesis by size, something that in the traditional happens via a

fake fence pair that encapsulates a strut of a certain size. In LMTX we use the same approach but have made the sequence more configurable. In practice that means that the values 1 up to 4 are just that but for some fonts we use the sequence 1 3 5 7. There was no need to adapt the engine as it already worked quite well.

Integrals

The Latin Modern fonts have only one size of big operators and one reason can be that there is no need for more. Another reason can be that there was just no space in the font. However, an OpenType font has plenty slots available and the reference font Cambria has integral signs in sizes as well as extensibles.

In LuaTeX we already have generic vertical extensibles but that only works well with specified sizes. And, cheating with delimiters has the side effect that we get the wrong spacing. In LuaMetaTeX however we have ways to adapt the size to what came or what comes. In fact, it is a mechanism that is available for any atom that we support. However, it doesn't play well with script and this whole `\limits` and `\nolimits` is a bit clumsy so Mikael and I decided that different route had to be followed. For adaptive large operators we provide this interface:

```
$ x + \integral [color=darkred,top={t},bottom={b}] {\frac{1}{x}} = 10 $
$ x + \startintegral [color=darkblue,top={t},bottom={b}]
  \frac{1}{x}
\stopintegral = 10 $
$ x + \startintegral [color=darkgreen,top={t},bottom={b},method=vertical]
  \frac{1}{x}
\stopintegral= 10 $
```

This text is not about the user interface so we won't discuss how to define additional large operators using one-liners.

The low level LuaMetaTeX implementation handles this input:

```
\Uoperator      \Udelimiter "0 \fam "222B {top} {bottom} {...}
\Uoperator limits \Udelimiter "0 \fam "222B {top} {bottom} {...}
\Uoperator nolimits \Udelimiter "0 \fam "222B {top} {bottom} {...}
```

plus the usual keywords that fenced accept, because after all, this is just a special case of fencing.

Currently these special left operators are implemented as a special case of fences because that mechanism does the scaling. It means that we need a (bogus) right fence, or need to brace the content (basically create an atom). When no right fence is found one is added automatically. Because there is no real fencing, right fences are removed when processing takes place. When you specify a `class` that one will be used for the left and right spacing, otherwise we have open/close spacing.

Going details

When the next feature was explored Mikael tagged it as math micro typography and the reason is that you need not only to set up the engine for it but also need to be aware of this kind of spacing. Because we wanted to get rid of this script spacing that the font imposes we configured ConTeXt with:

```
\setmathignore\Umathspacebefore\script\plusone
\setmathignore\Umathspaceafter\script \plusone
```

This basically nils all these tiny spaces. But the latest configuration has this instead:

```
% \setmathignore \Umathspacebefore\script\zerocount % default
% \setmathignore \Umathspaceafter\script \zerocount % default

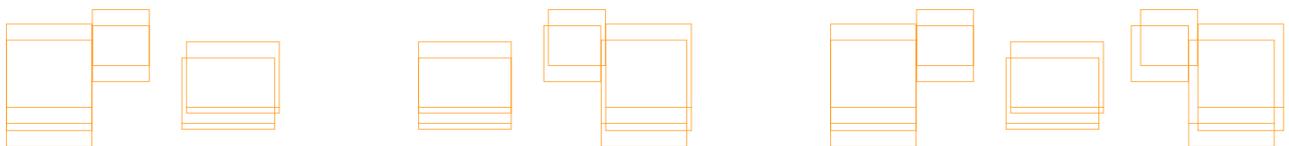
\mathslackmode \plusone

\setmathoptions\mathopcode \plusthree
\setmathoptions\mathbinarycode \plusthree
\setmathoptions\mathrelationcode\plusthree
\setmathoptions\mathopencode \plusthree
\setmathoptions\mathclosecode \plusthree
\setmathoptions\mathpunctcode \plusthree
```

This tells the engine to convert these spaces into what we call slack: disposable kerns at the edges. But it also converts these kerns into a glue component when possible. As with all these extensions it complicates the machinery but users will never see that. Now, the last six lines do the magic that made us return to honoring the spaces: we can tell the engine to ignore this slack when there are specific classes at the edges. These options are a bitset and **1** means “no slack left” and **2** means “no slack right” so **3** sets both.

```
\def\TestSlack#1%
  {\vbox\bgroup
    \mathslackmode\zerocount
    \hbox\bgroup
      \setmathignore\Umathspacebefore\script\zerocount
      \setmathignore\Umathspaceafter\script \zerocount
      #1
    \egroup
    \vskip-.9\lineheight
    \hbox\bgroup\red
      \setmathignore\Umathspacebefore\script\plusone
      \setmathignore\Umathspaceafter\script \plusone
      #1
    \egroup
  \egroup}

\startcombination[nx=3]
  {\showglyphs\TestSlack{\$f^2 > \$}} {}
  {\showglyphs\TestSlack{\$ > f^2\$}} {}
  {\showglyphs\TestSlack{\$f^2 > f^2\$}} {}
\stopcombination
```



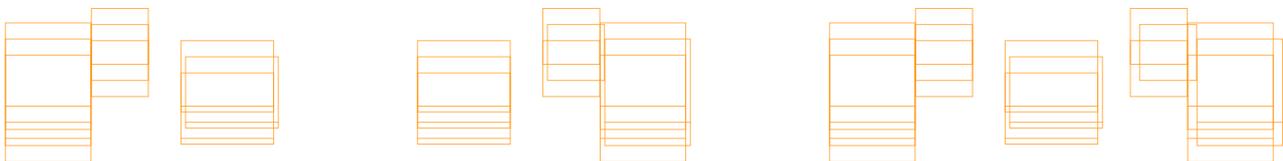
Because this overall removal of slack is not granular enough a while later we introduced a way to set this per class, as is demonstrated in the following example.

```

\def\TestSlack#1%
  {\vbox\bgroup
    \mathslackmode\plusone
    \hbox\bgroup\red
      \setmathignore\Umathspacebeforescript\zerocount
      \setmathignore\Umathspaceafterscript \zerocount
      #1
    \egroup
    \vskip-.9\lineheight
    \hbox\bgroup\green
      \setmathoptions\mathrelationcode \zerocount
      #1
    \egroup
    \vskip-.9\lineheight
    \hbox\bgroup\blue
      \setmathoptions\mathrelationcode \plusthree
      #1
    \egroup
  \egroup}

\startcombination[nx=3]
  {\showglyphs\TestSlack{\$f^2 > \$}} {}
  {\showglyphs\TestSlack{\$ > f^2\$}} {}
  {\showglyphs\TestSlack{\$f^2 > f^2\$}} {}
\stopcombination

```



Of course we need to experiment a lot with real documents and it might take a while before all this is stable (in the engine and in Con $\text{T}_{\text{E}}\text{X}$). And as we don't need to conform to the decades old golden $\text{T}_{\text{E}}\text{X}$ math standards we have some degrees of freedom in this: for Mikael and me it is pretty much a visual thing where we look closely at large samples. Of course in practice details get lost when we print at 10 point but that doesn't mean we can't provide the best experience.⁸

As we mention class specific options, we also need to mention the special case where we have for instance simple formulas like single atoms (for instance digits) are preceded by a sign (binary). These special spacing cases are handled by a lookahead flag that can be set `\setmathoptions <class>`, like the slack flags. More options might become available in due time. When set the lookahead will check for the automatically injected end class atom and use that for spacing when found. The mentioned lookahead is one of the hard coded heuristics in the traditional engine but here we need to explicitly configure it.

⁸ Whenever I look at (my) old (math) school books I realize that Don Knuth had very good reasons to come up with $\text{T}_{\text{E}}\text{X}$ and, it being hard to beat, $\text{T}_{\text{E}}\text{X}$ still sets the standard!

Ghosts

As plain T_EX has macros like `\vphantom` you also find them in macro packages that came later. These create a boxes that have their content removed after the dimensions are set. They take space and are invisible but there's also nothing there.

A variant in the upgraded math machinery are ghosts: these are visible in the sense that they show up but ignored when it comes to spacing. Here is an example. The option bit set here tells the engine that we ghost at the right, so we have ghosts around the relation (it controls where the spacing ends up).

```
$
x
\mathatom class \mathghostcode           {!!}
>
\mathatom class \mathghostcode options "00000020 {!!}
1
\quad
x
\mathatom class \mathghostcode           {\hbox{\smallinfofont ord}}
>
\mathatom class \mathghostcode options "00000020 {\hbox{\smallinfofont dig}}
1
$
```

You never know when this comes in handy but it fits in the new, more granular approach to spacing. The code above shows that it's just a class, this time with number 17.



Struts

In order to get consistent spacing the ConT_EXt macro package makes extensive use of struts in text mode as well as math mode. The normal way to implement that is either an empty box or a zero width rule, both with a specifically set height and depth. In ConT_EXt MkII and MkIV (and for a long time in LMTX too) they were rules so that we could visualize them: they get some width and kerns around them to compensate for that.

In order to not let them interfere with spacing we could wrap them into a ghost atom but it is kind of ugly. Anyway, before we had these ghost atoms an alternative to struts was already implemented: a special kind of rule. The reason is that I wanted a cleaner and more predictable way to adapt struts to the math style uses and sometimes predicting that is fragile. What we want is a delayed assignment of dimensions.

We have two solutions. The first one uses two math parameters that themselves adapt to the style, as do other parameters: `\Umathruleheight` and `\Umathruledepth`. The other solution relates a font (or family) and character with the strut rule which is then used as measure for the height and depth. Just for the record: this also works in text mode, which is why a recent LMTX also does use that for struts now. The optional visualization is just part of the regular visualization mechanism in ConT_EXt which already had provisions for struts. A side effect of this is that the rule primitives now accept three more keywords: `font`, `fam` and `char`, in addition to the already present traditional ones `width`, `height` and `depth`, the (backend) margin ones `left` (or `top`) and `right` (or `bottom`) options, as well as `xoffset` and `yoffset`). The command that creates

a rule with subtype `strut` is simply `\srule`. Because struts are rather macro package specific I leave it to this.

One positive side effect is that we could simplify the ConT_EXt fraction mechanism a bit. Over time control over the (font driven) gaps was introduced but that is not really needed because we zero the gaps anyway. There was also a tolerance mechanism which again was not used. However, for skewed fractions we do use the new tolerance mechanism as well as gap control.

Atoms

Now that we have generic atoms (`\mathatom`) another, sometimes confusing aspect of the math parsing can be solved. Take this:

```
\def\MyBin{\mathbin{\tt mybin}}
$ x ^ \MyBin _ \MyBin $
```

The parser just doesn't like that which means that one has to use

```
\def\MyBin{\mathbin{\tt mybin}}
$ x ^ {\MyBin} _ {\MyBin} $
```

or:

```
\def\MyBin{{\mathbin{\tt mybin}}}}
$ x ^ \MyBin _ \MyBin $
```

But the later has side effects: it creates a list that can influence spacing. It is for that reason that we do accept atoms where they were not accepted before. Of course that itself can have side effects but at least we don't get an error message. It fits well into the additional (user) classes model. And, given that in ConT_EXt the `\frac` command is actually wrapped as `\mathfrac` the next will work too:

```
$ x^{\frac{1}{2}} + x^{\frac{1}{2}} $
```

but in practice you should probably use the braced version here for clarity.

The vcenter primitive

Traditionally this primitive is bound to math but it had already been adapted to also work in text mode. As part of the upgrade of math we can now also pass all the options that normal boxed take and we can also cheat with the axis. Here is an example:

```
\def\TEST{\hbox\bgroup
  \darkred \vrule width 2pt height 4pt
  \darkgreen \vrule width 10pt depth 2pt
\egroup}
$
  x - \mathatom \mathvcentercode {!!!} -
  + \ruledvcenter \TEST
  + \ruledvcenter \TEST
  + \ruledvcenter axis 1 \TEST
  + \ruledvcenter xoffset 2pt yoffset 2pt \TEST
  + \ruledvcenter xoffset -2pt yoffset -2pt \TEST
```

+ x

\$

There was already a vcenter class available before we did this:

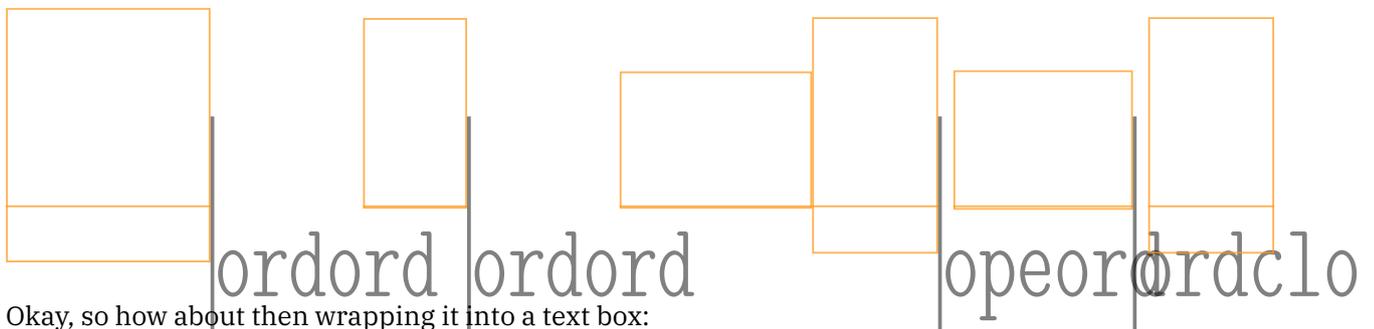


Text

Sometimes you want text in math, for instance `sin` or `cos` but text in math is not really text:

```
$ \setmathspacing\mathordordinarycode\mathordordinarycode\textstyle 10mu fin(x) $
```

The result demonstrates that what looks like a word actually becomes three math atoms:



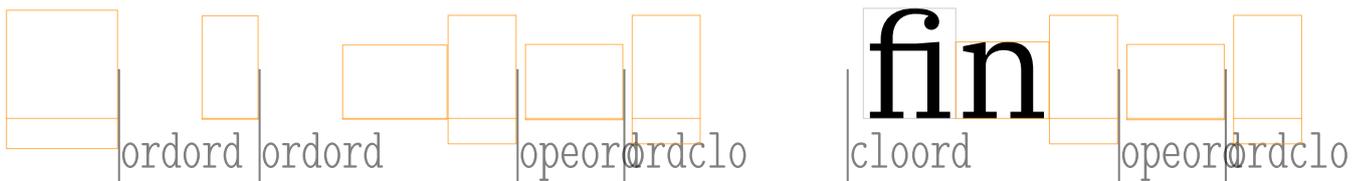
Okay, so how about then wrapping it into a text box:

\$

```
\setmathspacing\mathordordinarycode\mathordordinarycode\textstyle 10mu
fin(x) \quad \hbox{fin}(x)
```

\$

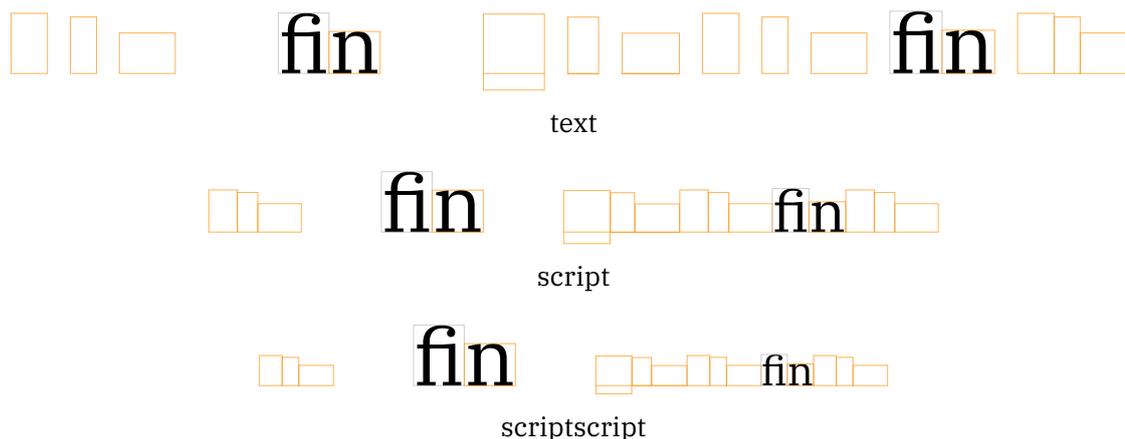
Here we get:



We even get a ligature which might be an indication that we're not using a math font which indeed is the case: the box is typeset in the regular text font.

```
\def\Test#1%
{\setmathspacing\mathordordinarycode\mathordordinarycode\textstyle 5mu
 $\showglyphs
 #1% style
 {\tf fin} \quad
 \hbox{fin} \quad
 \mathatom class \mathordordinarycode textfont {fin}
 \mathatom class \mathordordinarycode textfont {\tf fin}
 \mathatom class \mathordordinarycode textfont {\hbox{fin}}
 \mathatom class \mathordordinarycode mathfont {\hbox{fin}}
 $}
```

When we feed this macro with the `\textstyle`, `\scriptstyle` and `\scriptscriptstyle` we get:



Here you see a new atom option action: `textfont` which does as much as setting the font to the current family font and the size to the one used in the style. For the record: you only get ligatures when they are configured and provided by the font (and as math is a script itself it is unlikely to work).⁹

Tracing

I won't discuss the tracing features in ConTeXt here but for sure the visualizer helps a lot in figuring out all this. In LuaMetaTeX we carry a bit more information with the resulting nodes so we can provide more details, for instance about the applied spacing and penalties. Some is shown in the examples. A more recent tracing feature is this:

```
\tracingmath 1
\tracingonline 1
$
  \mathord (
  \mathord {({}
  \mathord \Udelimiter"4 0 ` (
  \Udelimiter"4 0 ` (
$
```

That gives us on the console (the dots represent detailed attribute info that we omit here):

```
7:3: > \inlinemath=
7:3: \noad[ord][...]
7:3: .\nucleus
7:3: ..\mathchar[open] family "0, character "28
7:3: \noad[ord][...]
7:3: .\nucleus
7:3: ..\mathlist
7:3: ... \noad[open][...]
7:3: .... \nucleus
7:3: ..... \mathchar[open] family "0, character "28
7:3: \noad[ord][...]
7:3: .\nucleus
```

⁹ The existing mechanisms in ConTeXt already dealt with this but it is nevertheless nice to have it as a clean engine feature.

```

7:3: ..\mathchar[open] family "0, character "28
7:3: \noad[open][...]
7:3: .\nucleus
7:3: ..\mathchar[open] family "0, character "28

```

A tracing level of 2 will spit out some information about applied spacing and penalties between atoms (when set) and level 3 will show the math list before the first and second pass (a mix of nodes and noads) we well as the result (nodes) plus return some details about rules, spacing and penalties applied.

Is there more?

The engine already provides the option to circumvent the side effect of a change in a parameter acting sort of global: the last value given is also the one that a second pass starts with. The `\frozen` prefix will turn settings into local ones but that's another (already old) topic. There are many such improvements and options not mentioned here but you can find them mentioned and explained in older development stories. A lot has been around for a while but not been applied in ConTeXt yet.

When TeX was written one important property (likely related to memory consumption) is that node lists have only forward pointers. That means that the state of preceding material has to be kept track of: there is no going (or looking) back. In LuaTeX we have double linked lists so in principle we can try to be more clever but so far I decided not to touch the math machinery in that way. But who knows what comes next.

Those italics

Right from the start of LuaTeX it became clear that the fact that TeX assumes the actual width of glyphs to be incremented by the italic correction that then selectively is removed has been an issue. It made for successive attempts to improve spacing in ConTeXt by providing pseudo features. But, when we moved from assembled Unicode math fonts to 'real' ones that became messy: what trick to apply when and even worse where? In the end there are only a very few shapes that actually are affected in the sense that when we don't deal with them it looks bad. It also happens that one of those shapes is the italic 'f', a letter that is used frequently in math. It might even be safe to say that the simple fact that the math italic f has this excessively wrong width and thereby pretty large italic correction is the cause of many problems.

In the LMTX approach Mikael and I settled on patching shapes in the so called font goodie files, aka `lfg` files and only a handful of entries needed a treatment. This makes a good case for removing the traditional font code path from LuaMetaTeX.

modern: $a_2^1 b_2^1 c_2^1 d_2^1 e_2^1 f_2^1 g_2^1 h_2^1 i_2^1 j_2^1 k_2^1 l_2^1 m_2^1 n_2^1 o_2^1 p_2^1 q_2^1 r_2^1 s_2^1 t_2^1 u_2^1 v_2^1 w_2^1 x_2^1 y_2^1 z_2^1$ $a_2^1 b_2^1 c_2^1 d_2^1 e_2^1 f_2^1 g_2^1 h_2^1 i_2^1$
 $j_2^1 k_2^1 l_2^1 m_2^1 n_2^1 o_2^1 p_2^1 q_2^1 r_2^1 s_2^1 t_2^1 u_2^1 v_2^1 w_2^1 x_2^1 y_2^1 z_2^1$

cambria: $a_2^1 b_2^1 c_2^1 d_2^1 e_2^1 f_2^1 g_2^1 h_2^1 i_2^1 j_2^1 k_2^1 l_2^1 m_2^1 n_2^1 o_2^1 p_2^1 q_2^1 r_2^1 s_2^1 t_2^1 u_2^1 v_2^1 w_2^1 x_2^1 y_2^1 z_2^1$ $a_2^1 b_2^1 c_2^1 d_2^1 e_2^1 f_2^1 g_2^1 h_2^1 i_2^1 j_2^1 k_2^1$
 $l_2^1 m_2^1 n_2^1 o_2^1 p_2^1 q_2^1 r_2^1 s_2^1 t_2^1 u_2^1 v_2^1 w_2^1 x_2^1 y_2^1 z_2^1$

pagella: $a_2^1 b_2^1 c_2^1 d_2^1 e_2^1 f_2^1 g_2^1 h_2^1 i_2^1 j_2^1 k_2^1 l_2^1 m_2^1 n_2^1 o_2^1 p_2^1 q_2^1 r_2^1 s_2^1 t_2^1 u_2^1 v_2^1 w_2^1 x_2^1 y_2^1 z_2^1$ $a_2^1 b_2^1 c_2^1 d_2^1 e_2^1 f_2^1 g_2^1 h_2^1 i_2^1 j_2^1 k_2^1 l_2^1 m_2^1$
 $n_2^1 o_2^1 p_2^1 q_2^1 r_2^1 s_2^1 t_2^1 u_2^1 v_2^1 w_2^1 x_2^1 y_2^1 z_2^1$

termes: $a_2^1 b_2^1 c_2^1 d_2^1 e_2^1 f_2^1 g_2^1 h_2^1 i_2^1 j_2^1 k_2^1 l_2^1 m_2^1 n_2^1 o_2^1 p_2^1 q_2^1 r_2^1 s_2^1 t_2^1 u_2^1 v_2^1 w_2^1 x_2^1 y_2^1 z_2^1$ $a_2^1 b_2^1 c_2^1 d_2^1 e_2^1 f_2^1 g_2^1 h_2^1 i_2^1 j_2^1 k_2^1 l_2^1 m_2^1 n_2^1$
 $o_2^1 p_2^1 q_2^1 r_2^1 s_2^1 t_2^1 u_2^1 v_2^1 w_2^1 x_2^1 y_2^1 z_2^1$

bonum: 

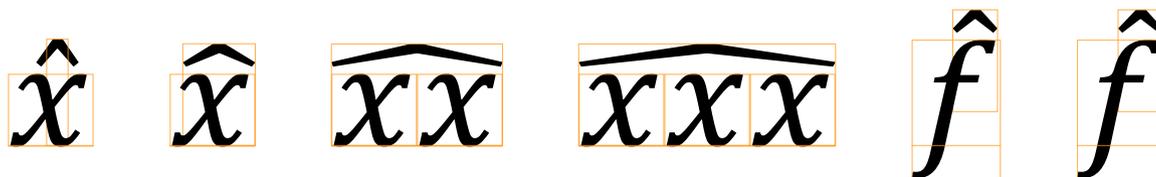
One of the other very sloped symbol is the integral, although most fonts have them more upright than tex has. Of course there are many variants of these integrals in a math font. Here we also have some font parameters that we can tune, which is what we do.

Accents

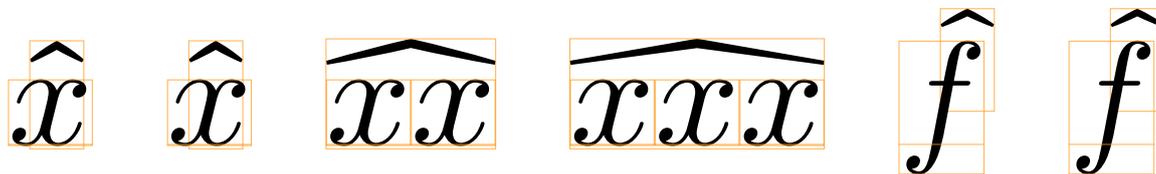
Accents are common in languages other than English and it's English that T_EX was made for. Although the seven bit variant became eight bit handling accents never was sophisticated and one of the main reasons is of course that one could use pre-built composed characters. The OpenType format brought proper anchoring (aka marks) to font formats and when LuaT_EX deals with text those kick in. In OpenType math however, anchoring is kind of limited to the top position only. Because the T_EX Gyre fonts are based on traditional T_EX fonts, their accents have not become better suited.

```
$ \hat{x} \enspace \widehat{x} \enspace \widehat{xx} \enspace \widehat{xxx}
  \enspace \hat{f} \enspace \widehat{f} $
```

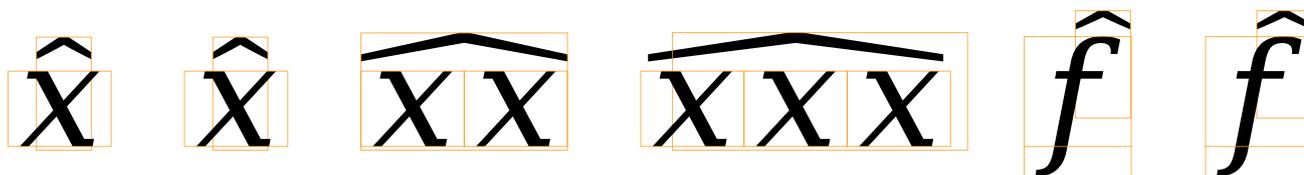
When looking at examples you need to be aware of the fact hat fonts can have been adapted in the goodie files.¹⁰ So, for instance bounding boxes and such can differ from the original. Anyway, the previous code in Cambria looks as follows.



With Latin Modern we get:



And Dejavu comes out as:



As you can see there are some differences. In for instance Latin Modern the shape of the hat and smallest wide hat are different and the first wide one has zero dimensions combined with a negative anchor. When an accented character is followed by a superscript or prime the italic correction of the base kicks in but that cannot be enough to not let this small wide hat overflow into the script. We could compensate for it but then we need to know the dimensions. Of course we can consult the bounding box but it makes no sense to let heuristics enter the machinery here while we're in the process generalization. One option is to have two extra parameters that can be used when the width of the accent comes close to the width of the base (we

¹⁰ Extreme examples can be found for Lucida Bright where we not only have to fix the extensible parts of horizontal braces but also have to provide horizontal brackets.

then assume that zero accent width means that it has base width) we add an additional kern. In the end we settled for a (semi automatic) correction option in the goodie files.

There are actually three categories of extensible accents to consider: those that resemble the ones used in text (like tildes and hats), those wrapping something (like braces and bracket but also arrows) and rules (that in traditional T_EX indeed are rules). In ConT_EXt we have different interfaces for each of these in order to have a more extensive control. The text related ones are the simplest and closest to what the engine supports out of the box but even there we use tweaked glyphs to get better spacing because (of course) fonts have different and inconsistent spacing in the boundingbox above and below the real shape. This is again some tweak that we moved from being *automatic* to being *under goodie file control*. But this is all too ConT_EXt specific to discuss here in more detail.

Decision time

After lots of tests Mikael and I came to the conclusion that we're facing the following situation. When typesetting math most single characters are italic and we already knew from the start of the LuaT_EX project that the italics shapes are problematic when it comes to typesetting math. But it looks like even some upright characters can have italic correction: in TexGyreBonum for instance the bold upright **f** has italic correction, probably because it then can (somehow) kern with a following **i**. It anyhow assumes no italic correction to be applied between these characters.

In the end the mixed math font model got more and more stressed so one decision was to simply assume fonts to be used that are either Cambria like OpenType, or mostly traditional in metrics, or a hybrid of both. It then made more sense to change the engine control options that we have into ones that simply enable certain code paths, independent of the fact if a font is OpenType or not. It then become a bit “crap in, crap out”, but because we already tweak fonts in the goodie files it's quite okay. Some fonts have bad metrics anyway or miss characters and it makes no sense to support abandoned fonts either. Also, when a traditional font is assembled one can set up the engine with different flags and we can deal with it as we wish. In the end it is all up to the macro package to configure things right, which is what we tried to do for months when rooting out all the artifacts that fonts bring.¹¹

That said, the reason why some (fuzzy) mixed model works out okay (also in LuaT_EX) is that proper OpenType fonts use staircase kerns instead of italic correction. They also have no ligatures and kerns. We also suspect that not that much attention is paid to the rendering. It's a bit like these “How many f's do you count in this sentence?” tests where people tend to overlook **of**, **if** and similar short words. Mathematicians loves **f**'s but probably also overlook the occasionally weird spacing and kerning.

A side effect is that mixing OpenType and traditional fonts is also no longer assumed which in turn made a few (newly introduced) state variables obsolete. Once everything is stable (including extensions discussed before) some further cleanup can happen. Another side effect is that one needs to tell the engine what to apply and where, like this:

```
\mathfontcontrol\numexpr \zerocount
  +\overrulemathcontrolcode
  +\underrulemathcontrolcode
  +\fractionrulemathcontrolcode
  +\radicalrulemathcontrolcode
  +\accentskewhalfmathcontrolcode
  +\accentskewappliedmathcontrolcode
```

¹¹ In previous versions one could configure this per font but that has been dropped.

```

% + checkligatureandkernmathcontrolcode
+ \applyverticalitalickernmathcontrolcode
+ \applyordinaryitalickernmathcontrolcode
+ \staircasekernmathcontrolcode
% + \applycharitalickernmathcontrolcode
% + \reboxcharitalickernmathcontrolcode
+ \applyboxeditalickernmathcontrolcode
+ \applytextitalickernmathcontrolcode
+ \checktextitalickernmathcontrolcode
% + \checkspaceitalickernmathcontrolcode
+ \applyscriptitalickernmathcontrolcode
+ \italicshapekernmathcontrolcode
\relax

```

There might be more control options (also for tracing purposes) and some of the symbolic (ConT_EXt) names might change for the better. As usual it will take some years before all is stable but because most users use the latest greatest version it will be tested well.

After this was decided and effective I also decided to drop the mapping from traditional font parameters to the OpenType derives engine ones: we now assume that the latter ones are set. After all, we already did that in ConT_EXt for the virtual assemblies that we started out with in the beginning of LuaT_EX and MkIV.

Dirty tricks

Once you start playing with edge cases you also start wondering if some otherwise complex things can be done easier. The next macro brings together a couple of features discussed in previous sections. It also uses two state variables: `\lastleftclass` and `\lastrightclass` that hold the most recent edge classes.

```

\tolerant\permanent\protected\def\NiceHack[#1]#:#2% special argument parsing
{\begingroup
  \setmathatomrule
  \mathbegincode\mathbincode % context constants
  \allmathstyles
  \mathbegincode\mathbincode
\normalexpanded
  {\setbox\scratchbox\hpack
   ymove \Umathaxis\Ustyle\mathstyle % an additional box property
   \bgroup
     \framed % a context macro
       [location=middle,#1]
       {\$\Ustyle\mathstyle#2$}%
     \egroup}%
\mathatom
  class 32 % an unused class
  \ifnum\lastleftclass <\zerocount\else leftclass \lastleftclass\fi
  \ifnum\lastrightclass<\zerocount\else rightclass \lastrightclass\fi
  \bgroup
    \box\scratchbox
  \egroup
\endgroup}

```

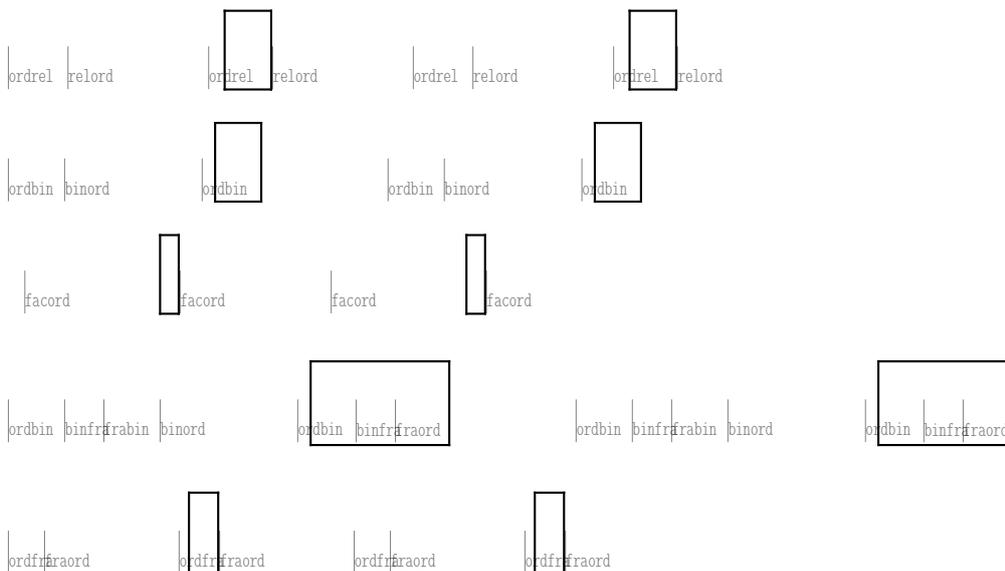
```

\def\MyTest#1%
  {$
    x #1
    $
    $\displaystyle x #1
    $\displaystyle x \NiceHack[offset=0pt]{#1} x $}

\scale[scale=2000]{\MyTest{>}} \blank
\scale[scale=2000]{\MyTest{+}} \blank
\scale[scale=2000]{\MyTest{!}} \blank
\scale[scale=2000]{\MyTest{+\frac{1}{2}+}} \blank
\scale[scale=2000]{\MyTest{\frac{1}{2}}} \blank

```

Of course this is not code you immediately come up with after reading this text, also because you need to know a bit of ConT_EXt.



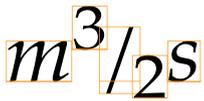
There are a few control options, like `\noatomruling` that can be used to prevent rules being applied to the next atom. We can use these in order to achieve more advanced alignment results, but discussing math alignments would demand many more pages than make sense here.

Tuned kerning

The ConT_EXt distribution has dedicated code for typesetting units that dates back to the mid nineties of the previous century but was (code wise) upgraded from MkII to MkIV which made it end up in the physics name space. There is not much reason to redo that code but when we talk new spacing classes it might make sense at some point to see if we can use less code for spacing by using a ‘unit’ class. When Mikael pointed out that, for instance in Pagella:



doesn't space well the obvious answer is: use the units mechanism because this kind of rendering was why it was made in the first place. However, the question is of course, can we do better anyway. The chosen solution uses a combination of class options and tweaked shape kerning:



An example of a class setup in ConT_EXt is:

```
\setmathoptions\mathdivisioncode\numexpr
  \nopreslackclassoptioncode      +\nopostslackclassoptioncode
  +\lefttopkernclassoptioncode    +\righttopkernclassoptioncode
  +\leftbottomkernclassoptioncode +\rightbottomkernclassoptioncode
\relax
```

and, although we don't go into the details of tweaking here, this is the kind of code you will find in the goodie file:

```
{
  tweak = "kerns",
  list = {
    [0x2F] = {
      topleft      = -0.3,
      bottomright =  0.2,
    }
  }
}
```

where the numbers are a percentage of the width. This specification translates in a math staircase kerning recipe.

More font tweaks

Once you start looking into the details of these fonts you are likely to notice more issues. For instance, in the nice looking Lucida math fonts the relations have inconsistent widths and even shapes. This can partially be corrected by using a stylistic alternate but even that forced us to come up with a mechanism to selectively replace 'bad' shapes because there is not that much granularity in the alternates. And once we looked at these alternates we noticed that the definition of of script versus calligraphic is also somewhat fuzzy and font dependent. That made for yet another tweak where we can swap alphabets and let the math machinery choose the expected shape. In Unicode this is handled by variant selectors which is rather cumbersome. Because these two styles are used mixed in the same document, a proper additional alphabet would have made more sense. As we already support variant selectors it was no big deal to combine that mechanism with a variant selector features over a range of calligraphic or script characters, which indeed is what mathematicians use (Mikael can be very convincing). With this kind of tweaks the engine doesn't really play a role: we always could and did deal with it. It's just that upgrading the engine made us look again at this.

Normalization

Once we had all these spacing related features upgraded it was time to move to other aspects math typesetting. Most of that is not handled in the engine but at the macro level. Examples of this are making sure that math spacing obeys the rules across alignment cells, breaking long formulas into lines with various alignment schemes. The first is accommodated by using the primitives that set the states at the beginning and end of a formula so that is definitely something that the engine facilitates. The second was already possible in MkIV but is somewhat more transparent now by using tagged boundary nodes.

But for this summary we stick to discussing the more low level features and where most of what we discussed here concerns horizontal spacing we also have some vertical magic like the mentioned scaled fences and operators but they sort of behave as expected given the traditional \TeX approach. We have some more:

```
\definemathradical[esqrt][sqrt][height=\maxdimen,depth=\maxdimen]
\definemathradical[ssqrt][sqrt][height=3ex,depth=2ex]
```

```
\def\TestSqrt#1%
  {test $ #1{x} + #1{\sin(x)} $ test\quad
  test $ #1{x} + #1{\sin(x)} + #1{\frac{1}{x}} $ test\quad
  test $ #1{x} + #1{x^2} $ test\quad
  test $ \left(#1{x} + #1{x^2} \right) $ test\par}
```

```
\TestSqrt \sqrt \blank
\TestSqrt \esqrt \blank
\TestSqrt \ssqrt \blank
```

```
test          test test          test test          test test          test
test          test test          test test          test test          test
test          test test          test test          test test          test
```

In the above example you see that square roots can be made to adapt themselves to other such roots. For this we had to add an additional pass. Originally there are just two passes: a first typesetting pass where the maximum height and depth are collected so that in the second pass the fences can be generated and injected. That second pass also handles the spacing and penalties. In LuaMeta \TeX we now have (1) radical body typesetting, (2) radical typesetting, (3) atom typesetting with height and depth analysis, (4) fence typesetting, and finally (5) inject spacing, penalties, remove slack, etc.

In the examples above we set the height and depth and these are passed by keywords to the radical primitive (most atoms and math structures accept keywords that control rendering). Here the special values `\maxdimen` signal that we have to make radicals of equal height and depth.

In MkII we had ways to snap formulas so that we got consistent line spacing. For a while I wondered if the engine could help with that but in the end no specific engine features are needed, but it is definitely an area that I keep an eye on because consistent spacing is important. After all one has to draw a line somewhere and we always have the Lua callback mechanism available.

More goodies

This summary will never be complete because we keep improving the rendering of math. For instance, when Mikael checked some less used math alphabets of Latin Modern and Bonum, as part of the goodie file completion, we were a bit horrified by the weird top accent anchoring, inconsistent dimensions and stale italic correction present in some glyphs. For instance there was a italic correction after an upright blackboard lowercase ‘f’, the upright digits had somewhat random top accent anchors, and due to the lack of granularity in for instance wide hats, characters that are often seen together got inconsistent wide hats. Also clashing with scripts was possible. All this resulted in yet another bunch of features:

- In the goodie files we added efficient options to remove anchors from alphabets (or individual characters).
- In the goodie files we added similar options to remove italic correction.
- Characters got a few extra fields: margins that can be used to cheat with dimensions so that we can get more consistent wide accents.
- The engine also got the possibility to compensate for accents when superscripts need to be anchored (by diminishing the height of accents as well as via an offsets).

We expect to add (and use) some more options like this when we run into other persistent issues. For sure there are some already that are not discussed here. Of course one can argue why we spend time on this: in 15 years of Unicode math usage in the T_EX community no one ever bothered about a wide hat over the digit 7 and no one wondered about the bad spacing after a lowercase blackboard f, but as we go on we do run into these phenomena and it has become a bit of an obsession to get it all right.¹²

By closely looking at default positioning of accents on top of characters Mikael noticed that the anchor points are actually always in the middle of the topmost left and right points of shapes. It looks like these points are calculates automatically and therefore you end up with an anchor on top of the highest part of the seven in Latin Modern Serif but in the middle of a seven with a flat top. You also get anchors at the top of the vertical line in b, d, and on the sticky bit of the g. It is a good example of being careful with automating font design. In our case, removing most anchors and adding a few later on was the solution.¹³

Untold stories

There are of course more features but not all make sense to discuss here. For instance, all hard coded properties are now configurable. Take for instance:

```
\Umathsuperscriptvariant\textstyle 1
\Umathsubscriptvariant \textstyle 1

$ 1_2^3 \quad {\scriptstyle 1_2^3} \quad {\displaystyle 1_2^3}$
```

This gives us:

Here the number refers to one of the build in variants, that themselves are a range of styles. In the next table the narrow variants are cramped:

0 normal	D	D	T	T	S	s	SS	ss
1 cramped	D	D	T	T	S	s	SS	ss
2 subscript	S	S	S	S	SS	SS	SS	SS
3 superscript	S	S	S	S	SS	SS	SS	SS

¹² Of course all this puts the usual bashing of Microsoft Word by users in a different perspective: limited control in the T_EX engine, faulty fonts that come with T_EX distributions, lack of testing and quality control, and probably the believe that all gets done well automatically plays a role here.

¹³ In the original fonts and traditional T_EX engine a kerning pair between a so called skew char and the character at hand is used.

4	small	S	S	S	S	SS	SS	SS	SS
5	smaller	S	s	S	s	SS	ss	SS	ss
6	numerator	S	s	S	s	SS	ss	SS	ss
7	denominator	s	s	s	s	SS	SS	SS	SS
8	double (superscript)	S	s	S	s	SS	ss	SS	ss

If you want you can change these values but of course we're then basically changing some of logic behind math rendering and for sure Don Knuth had good reasons for these defaults.

Another untold story relates to multi scripts. When a double script is seen, T_EX injects an ordinary recovery atom, issues an error message, and when told so just continues. In order to always continue LuaMetaT_EX introduces a mode variable that default to minus one, as negative values will trigger the error. Zero or positive values are interpreted as a class and bypass the error. Here is an example of usage:

```

\mathdoublescriptmode
  "\tohexadecimal\mathexperimentalcode % experimental class
  \tohexadecimal\mathexperimentalcode % we have to set both left and right

\setmathspacing \mathexperimentalcode \mathexperimentalcode \allmathstyles 20mu
\setmathspacing \mathordinarycode \mathexperimentalcode \allmathstyles 20mu

$x^1_2^3_4^5_6^7_8$

```

We get this:

Final words

One can argue that all these new features can make a document look better. But you only have to look at what Don Knuth produces himself to see that one always could do a good job with T_EX, although maybe at the cost of some extra spacing directives. It is the fact that OpenType showed up as well as many more math fonts, all with their own (sometimes surprising) special effects, that made us adapt the engine. Of course there are also new possibilities that permit better and more robust macro support. The T_EXbook has a chapter on “the fine points of mathematics typesetting” for a reason.

There has never been an excuse to produce bad looking documents. It is all about care. For sure there is a category of users who are forced to use T_EX, so they are excused. There are also those who have no eye for typography and rely on the macro package, so there we can to some extent blame the authors of those packages. And there are of course the sloppy users, those who don't enter a revision loop at all. They could as well use any system that in some way can handle math. One can also wonder in what way massive remote editing as well as collaborative working on documents make things better. It probably becomes less personal. At meetings and platforms T_EX users like to bash the alternatives but in the end they are part of the same landscape and when it comes to math they dominate. Maybe there is less to brag about then we like: just do your thing and try to do it as good as possible. Rely on your eyes and pay attention to the details, which is possible because the engine provided the means. The previous text shows a few things to pay attention to.

Now that all the basics that have to do with proper dimensions, spacing, penalties and logic are dealt with, we moved on to the more high level constructs. We also haven't applied some features in the ConT_EXt code base yet and are now experimenting with the more high level constructs. For instance the frequently used

math alignment mechanism has been overhauled to support advanced inter atom spacing across rows, and in practice one will now more often not even use this alignment mechanism and use the alignment features in multi-line display math, if only because they offer advanced annotation. As a nice side effect some of the mechanism that we use for this (like the improved `\vadjust` primitive engine feature) also became somewhat more powerful in regular text mode and we'll see where that brings us.

Given the time we spend on this and given the numerous new features it will take a while before all that got added to the engine will be documented. Of course the usage in ConT_EXt also serves as documentation. This is not really a problem because most users will happily rely on the goodie files being okay and maintained, and usage other than ConT_EXt is unlikely to use these new features, if only because it will break away from the established long term standards and habits.

6 The binary

This is a very short chapter. Because LuaMetaTeX is also a script runner, I want to keep it lean and mean. So, when the size exceeded 3MB after we'd extended the math engine, I decided to (finally) let all the MetaPost number interfaces pass pointers which brought down the binary 100K and below the 3MB mark again.

I then became curious about how much of the binary actually is taken by MetaPost, and a bit of calculation indicated that we went from 20.1% down to 18.3%. Here is the state per May 13, 2022:

component	pct	bytes	comment
liblua	11.8	349158	lua core, tex interfaces
libluaoptional	2.4	70263	framework, several small interfaces, cerf
libluarest	1.9	55911	general helper libraries
libluasocket	2.4	71640	helper that interfaces to the os libraries
libmimalloc	4.1	121186	memory management partial
libminiz	1.2	34962	minimalistic core
libmp	18.3	540615	mp graphic core, number libraries, lua interfacing
libpplib	7.4	220386	pdf reading core, encryption helpers
libtex	50.5	1495970	extended tex core
luametatex		2960091	2022-05-13

It is clear that the TeX core is good for half of the code (50.5%) with the accumulated Lua stuff (18.5%) and MetaPost being a good second (18.3%) and third and the pdf interpreting library a decent fourth (7.4%) place.

7 To the point

In the 2022 ntg Maps 53 there is a visual very attractive article about generative graphics with MetaPost by Fabrice Larribe. These graphics actually use very little MetaPost code that use randomized paths and points and the magic is in getting the parameters right. This means that one has to process them a lot to figure out what looks best. Here is an example of such a graphic definition. I will show more variants so a rendering happens later on.

```
\startMPdefinitions
  vardef agitate_a(expr thepath, S, n, fn, t, ft) =
    save R, nbpoints, noiselevel, rlength ;
    path R ; nbpoints := n ; noiselevel := t ;
    R := thepath ;
    for s=1 upto S :
      nbpoints := nbpoints * fn ;
      noiselevel := noiselevel * ft ;
      R := for i=1 upto nbpoints:
        point (i/nbpoints) along R
          randomized noiselevel
        ..
      endfor cycle ;
    endfor ;
  R
enddef ;
\stopMPdefinitions
```

I will not explain the working of this because there is the article. Instead, I will focus on something that came up when the Maps was prepared: performance. Not only are these graphics large (which is no real problem) but they also take a while to render (which is something that does matter when one wants to find the best a parameters). For the first few variants we keep the same names of variables as in the article.

In figure 7.1 we show the (kind of) graphic that we are dealing with. Such an agitator is used in a loop so that we agitate multiple circles, where we go from large to small with for instance 4868, 4539, 4221, 3892, 3564, 3245, 2917, 2599, 2270, 1941, 1623, 1294, 966, 647 and 319 points. The article uses a definition like below for the graphic where you can see the agitator being applied to each of the circles.

```
path P ; numeric NbCircles, S, nzero, fn, tzero, ft ;

randomseed := 10 ;
defaultscale := .05 ;

NbCircles := 15 ; S := 10 ; nzero := 10 ; fn := 1.3 ; tzero := 5 ; ft := 0.8 ;

for c = NbCircles downto 1 :
  P := fullcircle scaled (c*6.5) scaled 3 ;
  P := agitate_a(P, S, nzero, fn, tzero, ft) ;
  eofill P
  withcolor transparent(1,4/NbCircles,col) ;
draw P
  withpen pencircle scaled 0.1
```

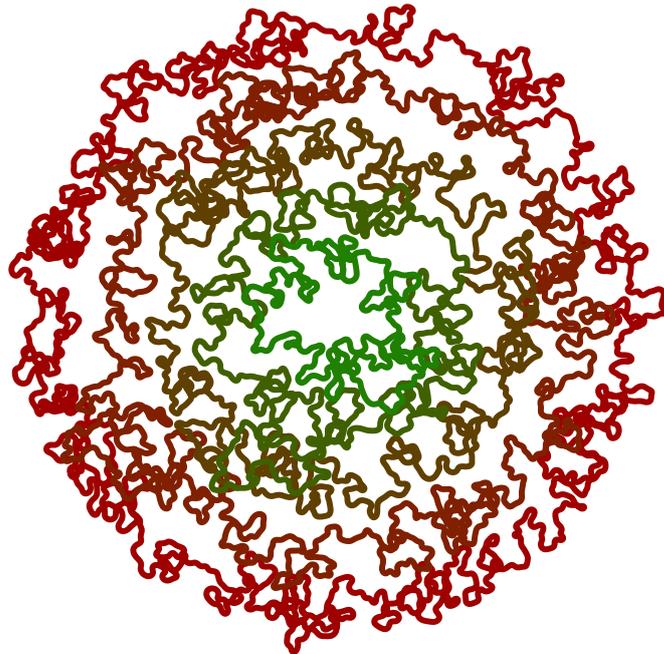


Figure 7.1 Fabrice's agitated circles, with reduced properties to keep this file small (see source).

```
transparent(1,4/NbCircles,.90[black,col]) ;
endfor ;
```

The first we noticed is that the graphics processes faster when double mode is used: we gain 40–50% and the reason for this is that modern processors are very good at handling doubles while MetaPost in scaled mode has to do a lot of juggling with pseudo fractions. In the timings shown later we leave that improvement out. Also, because of this observation ConT_EXt LMTX now defaults its MetaPost instances to method double.

When I stared at the agitator code I noticed that the `along` macro was used. That macro returns a point at given percentage along a path. In order to do that the macro calculates the length of the path and then locates that point. The primitive operations involved are `arclength`, `arctime of` and `point of` and each these takes some time to complete. A first improvement is to inline the `along` and hoist the length calculation outside the loop.

```
\startMPdefinitions
  vardef agitate_b(expr thepath, S, n, fn, t, ft) =
    save R, nbpoints, noiselevel, rlength ;
    path R ; nbpoints := n ; noiselevel := t ;
    R := thepath ;
    for s=1 upto S :
      nbpoints := nbpoints * fn ;
      noiselevel := noiselevel * ft ;
      rlength := (arclength R) / nbpoints;
      R := for i=1 upto nbpoints:
        (point (arctime (i * rlength) of R) of R)
          randomized noiselevel
      ..
    endfor cycle ;
  endfor ;
R
```

```

    enddef ;
\stopMPdefinitions

```

There is not that much that we can improve here but because Mikael Sundqvist and I had just extended MetaPost with some intersection improvements, it made sense to see what we could do in the engine. In the next variant the `arcpoint` combines `arctime of` and `point of`. The reason this is much faster is that we are already on the right spot when we got the time, and we save a sequential `point of` lookup, something that takes more time when paths are longer.

```

\startMPdefinitions
  vardef agitate_c(expr thepath, S, n, fn, t, ft) =
    save R, nbpoints, noiselevel, rlength ;
    path R ; nbpoints := n ; noiselevel := t ;
    R := thepath ;
    for s=1 upto S :
      nbpoints := nbpoints * fn ;
      noiselevel := noiselevel * ft ;
      rlength := (arclength R) / nbpoints;
      R := for i=1 upto nbpoints:
        (arcpoint (i * rlength) of R)
          randomized noiselevel
      ..
    endfor cycle ;
  endfor ;
  R
enddef ;
\stopMPdefinitions

```

At that stage we wondered if we could come up with a primitive like `intersectiontimelist` for these points; here a list refers to a path in which we collect the points. Now, as with the intersection primitives, MetaPost loops over the segments of a path and works within such a segment. That is why the following variant has an explicit start at point zero: we can now use offsets (discrete points).

```

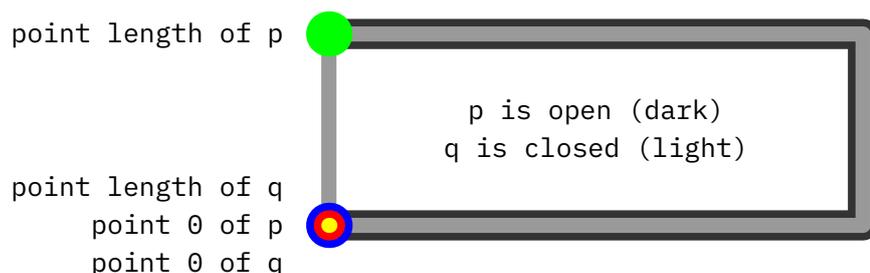
\startMPdefinitions
  vardef agitate_d(expr thepath, S, n, fn, t, ft) =
    save R, nbpoints, noiselevel, rlength ;
    path R ; nbpoints := n ; noiselevel := t ;
    R := thepath ;
    for s=1 upto S :
      nbpoints := nbpoints * fn ;
      noiselevel := noiselevel * ft ;
      rlength := (arclength R) / nbpoints;
      R := for i=1 upto nbpoints:
        (arcpoint (0, i * rlength) of R)
          randomized noiselevel
      ..
    endfor cycle ;
  endfor ;
  R
enddef ;
\stopMPdefinitions

```

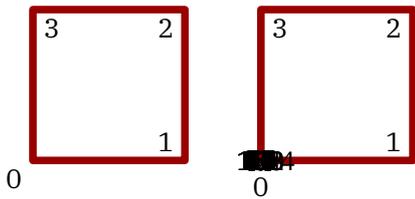
During an evening zooming Mikael and I figured out, by closely looking at the source, how the arc functions work and how we could indeed come up with a list primitive. The main issue was to use the right information. Mikael sat down to make a pure MetaPost variant and I hacked the engine. Mikael came up with a first variant similar to the following, where we use a new primitive `subarclength`.

```
\startMPdefinitions
  vardef arcpoints_a(expr thepath, cnt) =
    save len, seg, tot, tim, stp, acc ;
    numeric len ; len := length thepath ;
    numeric seg ; seg := 0 ;
    numeric tot ; tot := 0 ;
    numeric tim ; tim := 0 ;
    %
    numeric acc[] ; acc[0] := 0 ;
    for i = 1 upto len:
      acc[i] := acc[i-1] + subarclength (i-1,i) of thepath ;
    endfor;
    %
    numeric stp ; stp := acc[len] / cnt;
    %
    point 0 of thepath
    for tot = stp step stp until acc[len] :
      hide(
        forever :
          exitif ((tim < tot) and (tot < acc[seg+1])) ;
          seg := seg + 1 ;
          tim := acc[seg] ;
        endfor ;
      )
      -- (arcpoint (seg,tot-tim) of thepath)
    endfor if cycle thepath : -- cycle fi
  enddef ;
\stopMPdefinitions
```

Getting points of a path is somewhat complicated by the fact that the length of a closed path is different from that of an open path even if they have the same number of so-called knots. Internally a path is always a closed loop. That way, when MetaPost runs over a path, it can easily access the first point when it's at the end, something that is handy when that point has to be taken into account. Therefore, the end condition of a loop over a path is the arrival at the beginning. In the next graphic we show a bit how these first (zero) and last points are located. One reason why the previous macros start at point one and not at zero is that `arclength` can overflow due to the randomly growing path otherwise.



The difference between starting at zero or one for a cycle is show below, we get more and more points!



In the next variants we will not loop over points but step to the `arclength`. Watch the new `subarclength` primitive that starts at an offset. This is much faster than taking a `subpath of`. We can move the accumulator loop into the main loop:

```
\startMPdefinitions
  vardef arcpoints_b(expr thepath, cnt) =
    save len, aln, seg, tot, tim, stp, acc ;
    numeric len ; len := length thepath ;
    numeric aln ; aln := arclength thepath ;
    numeric seg ; seg := 0 ;
    numeric tot ; tot := 0 ;
    numeric tim ; tim := 0 ;
    numeric stp ; stp := aln / cnt;
    numeric acc ; acc := subarclength (0,1) of thepath ;
    %
    point 0 of thepath
    for tot = stp step stp until aln :
      hide(
        forever :
          exitif tot < acc ;
          seg := seg + 1 ;
          tim := acc ;
          acc := acc + subarclength (seg,seg+1) of thepath ;
        endfor ;
      )
      -- (arcpoint (seg,tot-tim) of thepath)
    endfor if cycle thepath : -- cycle fi
  enddef ;
\stopMPdefinitions
```

If you don't like the `hide` the next variant also works okay:

```
\startMPdefinitions
  vardef mfun_arc_point(text tot)(text thepath) =
    forever :
      exitif tot < acc ;
      seg := seg + 1 ;
      tim := acc ;
      acc := acc + subarclength (seg,seg+1) of thepath ;
    endfor ;
    (arcpoint (seg,tot-tim) of thepath)
  enddef ;

  vardef arcpoints_c(expr thepath, cnt) =
    save len, aln, seg, tot, tim, stp, acc ;
```

```

numeric len ; len := length thepath ;
numeric aln ; aln := arclength thepath ;
numeric seg ; seg := 0 ;
numeric tot ; tot := 0 ;
numeric tim ; tim := 0 ;
numeric stp ; stp := aln / cnt;
numeric acc ; acc := subarclength (0,1) of thepath ;
%
point 0 of thepath
for tot = stp step stp until aln :
  -- mfun_arc_point(tot)(thepath)
endfor if cycle thepath : -- cycle fi
enddef ;
\stopMPdefinitions

```

This got applied in three test agitators

```

\startMPdefinitions
vardef agitate_e_a(expr thepath, S, n, fn, t, ft) =
  save R, nbpoints, noiselevel ;
  path R ; nbpoints := n ; noiselevel := t ;
  R := thepath ;
  for s=1 upto S :
    nbpoints := nbpoints * fn ;
    noiselevel := noiselevel * ft ;
    R := arcpoints_a(R, nbpoints) ; % original Mikael
    R := for i=0 upto length R:
      (point i of R)
      randomized noiselevel
    ..
  endfor cycle ;
endfor ;
R
enddef ;

vardef agitate_e_b(expr thepath, S, n, fn, t, ft) =
  save R, nbpoints, noiselevel ;
  path R ; nbpoints := n ; noiselevel := t ;
  R := thepath ;
  for s=1 upto S :
    nbpoints := nbpoints * fn ;
    noiselevel := noiselevel * ft ;
    R := arcpoints_b(R, nbpoints) ; % merged Mikael
    R := for i=0 upto length R:
      (point i of R)
      randomized noiselevel
    ..
  endfor cycle ;
endfor ;
R

```

```

enddef ;

vardef agitate_e_c(expr thepath, S, n, fn, t, ft) =
  save R, nbpoints, noiselevel ;
  path R ; nbpoints := n ; noiselevel := t ;
  R := thepath ;
  for s=1 upto S :
    nbpoints := nbpoints * fn ;
    noiselevel := noiselevel * ft ;
    R := arcpoints_c(R, nbpoints) ; % split Mikael
    R := for i=0 upto length R:
      (point i of R)
        randomized noiselevel
      ..
    endfor cycle ;
  endfor ;
  R
enddef ;
\stopMPdefinitions

```

The new engine primitive shortens these agitators:

```

\startMPdefinitions
vardef agitate_e_d(expr thepath, S, n, fn, t, ft) =
  save R, nbpoints, noiselevel ;
  path R ; nbpoints := n ; noiselevel := t ;
  R := thepath ;
  for s=1 upto S :
    nbpoints := nbpoints * fn ;
    noiselevel := noiselevel * ft ;
    R := arcpointlist nbpoints of R;
    R := for i=0 upto length R:
      (point i of R)
        randomized noiselevel
      ..
    endfor cycle ;
  endfor ;
  R
enddef ;
\stopMPdefinitions

```

So are we done? Did we get rid of all bottlenecks? The answer is no! We still loop over the list in order to randomize the points. For each point we start at the beginning of the list. Let's first rewrite the agitator a little:

```

\startMPdefinitions
vardef agitate_f_a(expr pth, iterations, points, pointfactor, noise,
noisefactor) =
  save currentpath, currentpoints, currentnoise ; path currentpath ;
  currentpath := pth ;
  currentpoints := points ;

```

```

currentnoise := noise ;
for step = 1 upto iterations :
    currentpath := arcpointlist currentpoints of currentpath ;
    currentnoise := currentnoise * noisefactor ;
    currentpoints := currentpoints * pointfactor ;
    if currentnoise <> 0 :
        currentpath :=
            for i = 0 upto length currentpath:
                (point i of currentpath) randomized currentnoise ..
            endfor
        cycle ;
    fi
endfor ;
currentpath
enddef ;
\stopMPdefinitions

```

One of the LuaMetaFun extensions is a fast path iterator. In the next variant the `inpath` macro sets up an iterator (regular loop) with the length as final value. In the process the given path gets passed to Lua where we can access it as array. The `pointof` macro (again a Lua call) injects a pair. You will be surprised that even with passing the path to Lua and calling out to Lua to inject the pair this is way faster than the built-in `point of`.

```

\startMPdefinitions
vardef agitate_f_b(expr pth, iterations, points, pointfactor, noise,
noisefactor) =
    save currentpath, currentpoints, currentnoise ; path currentpath ;
    currentpath := pth ;
    currentpoints := points ;
    currentnoise := noise ;
    for step = 1 upto iterations :
        currentnoise := currentnoise * noisefactor ;
        currentpoints := currentpoints * pointfactor ;
        currentpath := arcpointlist currentpoints of currentpath ;
        if currentnoise <> 0 :
            currentpath :=
                for i inpath currentpath :
                    (pointof i) randomized currentnoise ..
                endfor
            cycle ;
        fi
    endfor ;
    currentpath
enddef ;
\stopMPdefinitions

```

It was tempting to see if a more native solution pays off. One problem there is that a path is not really suitable for that as we currently don't have a data type that represents a point. Okay, actually we sort of have because we can use the transform record that has six points but that is something I will look into later (it just got added to the todo list).

The `i within pth` iterator is no conceptual beauty but does the job. Just keep in mind that it is just means for this kind of applications: run over a path point by point. The `i` has the current point number. Because we run over a path following the links we only run forward.

```

\startMPdefinitions
  vardef agitate_f_c(expr pth, iterations, points, pointfactor, noise,
    noisefactor) =
    save currentpath, currentpoints, currentnoise ; path currentpath ;
    currentpath := pth ;
    currentpoints := points ;
    currentnoise := noise ;
    for step = 1 upto iterations :
      currentnoise := currentnoise * noisefactor ;
      currentpoints := currentpoints * pointfactor ;
      currentpath := arcpointlist currentpoints of currentpath ;
      if currentnoise <> 0 :
        currentpath :=
          for i within currentpath :
            pathpoint
              randomized currentnoise
            ..
          endfor
        cycle ;
      fi
    endfor ;
    currentpath
  enddef ;
\stopMPdefinitions

```

Any primitive solution more complex than this, like first creating a fast access data structure, of having a double linked list, or using some iterator larger than a simple numeric is very likely to have no gain over the super fast Lua variant.

We show the average runtime for three runs. Here we don't render the paths, which takes about one second, including conversion to pdf. Of course measurements like this can change a bit over time. To these times you need to add about a second for the draw and fill operations as well as conversion to a pdf stream with transparencies. The improvement in runtime makes it possible to use agitators like this at runtime especially because normally one will not use such (combinations of) large paths.

agitate_a	776.26	agitate_e_a	291.99	agitate_f_a	10.82
agitate_b	276.43	agitate_e_b	76.06	agitate_f_b	2.55
agitate_c	259.89	agitate_e_c	77.27	agitate_f_c	2.17
agitate_d	260.41	agitate_e_d	18.67		

The final version of the agitator is slightly different because it depends if we start at zero or one but gives similar results and adapt the noise before or after the loop.

```

\startMPdefinitions
  vardef agitator(expr pth, iterations, points, pointfactor, noise, noisefactor)
  =
    save currentpath, currentpoints, currentnoise ; path currentpath ;

```

```

currentpath := pth ;
currentpoints := points ;
currentnoise := noise ;
for step = 1 upto iterations :
    currentpath := arcpointlist currentpoints of currentpath ;
    if currentnoise <> 0 :
        currentpath :=
            for i within currentpath :
                pathpoint
                    randomized currentnoise
                ..
            endfor
        cycle ;
    fi
    currentnoise := currentnoise * noisefactor ;
    currentpoints := currentpoints * pointfactor ;
endfor ;
currentpath
enddef ;
\stopMPdefinitions

```

We use a similar example as in the mentioned article but coded a bit differently:

```

\startMPcode
path pth ;
nofcircles := 15 ; iterations := 10 ;
points := 10 ; pointfactor := 1.3 ;
noise := 5 ; noisefactor := 0.8 ;

nofcircles := 5 ; iterations := 10 ;
points := 5 ; pointfactor := 1.3 ;

% for c = nofcircles downto 1 :
% pth := fullcircle scaled (c * 6.5) scaled 3 ;
% points := floor(arclength(pth) * 0.5) ;
% pth := agitator(pth, iterations, points, pointfactor, noise, noisefactor) ;
% eofill pth
% withcolor darkred
% withtransparency(1,4/nofcircles) ;
% draw pth
% withpen pencircle scaled 0.1
% withtransparency(1,4/nofcircles) ;
% endfor ;

% currentpicture := currentpicture x sized TextWidth ;

for c = nofcircles downto 1 :
    pth := fullcircle scaled (c * 6.5) scaled 3 ;
    points := floor(arclength(pth) * 0.5) ;
    pth := agitator(pth, iterations, points, pointfactor, noise, noisefactor) ;
    draw pth

```

```
    withpen pencircle scaled 1
    withcolor (c/nofcircles)[darkgreen,darkred] ;
endfor ;

currentpicture := currentpicture xsize .5TextWidth ;
\stopMPcode
```

For Mikael and me, who both like MetaPost, it was a nice distraction from working months on extending math in LuaMetaTeX, but it also opens up the possibilities to do more with rendering (math) functions and graphics, so in the end we get paid back anyway.

8 Not all makes sense

The development of ConT_EXt is to a large extent driven by users with a wide variety of background and usage. I can safely say that much time spent on ConT_EXt qualifies as hobby (or maybe even more by curiosity). Of course I do use it myself but personally I never make advanced documents. I'm not a writer, nor an artist, nor a typesetter. I do like challenges so that's why we get mechanisms that can do tricky things and some stay sort of hidden because the practical usage is limited, although you will be surprised to see what users find in the source and use anyway. My colleague uses ConT_EXt for large scale, mostly complex and demanding xml documents where one source is rendered in different ways with different parts used. Many features in ConT_EXt relate to workflows.

I like to visualize things so that's part of the development cycle. I never start from some 'typographical' point of view, if only because in my experience much design is arbitrary and personal. The output should look okay on the average, and on reasonable simple documents there should be no need for manual intervention. I am quite willing to accept an occasional less optimal looking page and don't lose sleep over it. A next time, when a sentence gets added, it might be better and the problem can be moved further down the pages. Also, given what one runs into nowadays the average job that T_EX does is pretty good (but users can of course mess up). It is boundary conditions that determine in what direction a style or solution goes. The more abstract one argues about typesetting and possible solutions, the less interested I often become simply because there are no perfect solutions for every case. There are always those last few % points that need manual intervention or some trickery and most users get that. It is also what makes using T_EX fun.

As mentioned, the T_EX engine does a pretty good job on average but that didn't prevent me from extending it: the mix of T_EX, MetaPost and Lua is even more fun. But what is the development agenda there? Again, it is very much driven by what users want me to solve, but there's also the curiosity element. A recent example of extending is the math sub system. It was already made more configurable and some features were added but now it is really flexible. This was doable because the heuristics in the engine are clear. It was could be done because I had a dedicated partner in this journey.¹⁴ Other parts are more difficult but have nevertheless been extended, to mention a few: alignments, par building and page building. However the last two use some heuristics that are hard to make more flexible. For instance the badness calculation combined with the loop that tries to find breakpoints is already quite good and the somewhat special values involved in the calculations have been optimized stepwise by Don Knuth during the development of T_EX.

Does that mean that one cannot add some options to influence that tuning? For sure one can. The source has this comment:

“When looking for optimal line breaks, T_EX creates a ‘break node’ for each break that is *feasible*, in the sense that there is a way to end a line at the given place without requiring any line to stretch more than a given tolerance. A break node is characterized by three things: the position of the break (which is a pointer to a `glue_node`, `math_node`, `penalty_node`, or `disc_node`); the ordinal number of the line that will follow this breakpoint; and the fitness classification of the line that has just ended, i.e., `tight_fit`, `decent_fit`, `loose_fit`, or `very_loose_fit`.”

The book T_EX by Topic (by Eijkhout) gives a good explanation of the way lines are broken so there is no need to go into detail here. The code involved is not that trivial anyway. The criteria for deciding what is bad are as follows:

verdict	effect	badness
---------	--------	---------

¹⁴ In another chapter I summarize what Mikael Sundqvist and I did in this context.

very loose	stretch	>= 100
loose	stretch	>= 13
decent		<= 12
tight	shrink	>= 13

When the difference between two lines is more than one, they are considered to be visually incompatible. Then, if the badness of any line exceeds `pretolerance` a second pass is triggered, When `pretolerance` is negative the first pass is skipped. When the badness of any line exceeds `tolerance` a third pass is triggered and `emergencystretch` is used to make things fit.

Where in traditional \TeX a lot of parsing, hyphenation, font handling and par building is combined, in Lua-Meta \TeX we always work with completely hyphenated and font readied lists. In traditional \TeX the first pass works on the original non-hyphenated lists.

In the source there is an old note that one day I will play with a plugged in badness calculation but it also says that there might be a performance impact as well as all kind of unforeseen side effects because \TeX makes sure that the heuristics lead to values that don't result in overflow and such.

Another note concerns more fitness values. Doing that will increase the runtime a little but on a modern machine that is not really an issue. Shortly after I upgraded my laptop to a somewhat newer one I decided to play with this and therefore any performance hit would go unnoticed anyway. The following snippet from the source shows the idea:

```
typedef enum fitness_value {
    very_loose_fit, /*tex lines stretching more than their stretchability */
    loose_fit,      /*tex lines stretching 0.5 to 1.0 of their stretchability */
    semi_loose_fit,
    decent_fit,     /*tex for all other lines */
    semi_tight_fit,
    tight_fit,      /*tex lines shrinking 0.5 to 1.0 of their shrinkability */
    n_of_fitness_values
} fitness_value;
```

This means that when we loop over `very_loose_fit` upto `tight_fit` we have two more classes to take into account: the semi ones. Playing with that and associating them with magic numbers quickly learned that we enter the area of ‘random improvements’. You can render variants and because some will look better and others worse one can argue for any case. And as usual, once a user (unaware of what we are doing) looks at it, things like successive hyphens, wider spaces, rivers and such are seen as the main difference. Of course spacing is the direct result of this kind of messing, but because the effects are actually mostly noticeable on non-justified texts it then is the end-of-line spacing that influences the verdict.¹⁵

In the end this kind of extensions make little sense. One can of course play science and introduce all kind of imaginary cases where it might work but that is why I started this summary by explaining what drives developments: users and constraints. Playing science for the sake of it is pseudo science. And, as with

¹⁵ When hz showed up in pdf \TeX we did experiments with random samples of its usage and \TeX ies at user group meetings and the results were such that one could only draw the conclusion that on the average a user has no clue if something is good or bad for what reason. The strong emphasis in the \TeX community on hyphenation makes that an eye-catching criterium. So having two in a successive lines even when there is really no better solution is what draws the attention and users then tend to think that what a survey is about is “The quality of hyphenation related to breaking paragraphs into lines.”

much science related to typesetting (probably with the exception of Don's work) most has therefore little practical value.

So, do we keep this feature or not? We actually do, if only to be able to demonstrate the fuzziness of this. We have an undocumented magic parameter:

```
\linebreakcriterium"0C0C0C63
```

Actually the value is zero but when one of the four byte pairs is zero it will default to "0C (12) or "63 (99). The values concern `semitight`, `decent`, `semiloose`, and `loose`. After some trial and error I got to the examples on the next two pages. You need to zoom in to see the differences (the black one is the original). In setting used are:

```
\hsize \setupalign
```

- 1 12em normal, stretch, tolerant
- 2 18em flushleft

As mentioned, one can look at specific expected properties and draw conclusions but when T_EX cannot find a good solution using its default, it is unlikely that alternative settings help you out, unless you do that on a per-paragraph basis.

9 But this does

In LuaMetaT_EX one can do a lot on Lua, like what I will discuss next, but because it is somewhat fundamental it became a core feature of the engine. It was also quite easy to implement. It has to do with packaging.¹⁶

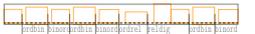
The box constructors in traditional T_EX accept two keywords: `to` for setting an exact width and `spread` for specifying additional width. In LuaMetaT_EX we have some more like `shift` (a traditional T_EX concept), `orientation`, `xmove`, `xoffset`, `ymove` and `yoffset` for absolute positioning, `anchor(s)`, `target` and `source` for relative positioning, `axis` and `class` for usage in `delay` for leader like boxes, the multiple `attr` key for setting attributes, a special subtype directive `container` and `direction` for controlling bidirectional typesetting, and `reverse` for reversing content. The latest addition: `adapt` is there for controlling and freezing glue.

So, in addition to the width related keys `to` and `spread` we have `adapt` that drives wo what width the box will be typeset.¹⁷ The keyword is followed by a scale values between -1000 and 1000 where a negative value enforces shrink and a positive value stretch. The following table shows the effects:

	<code>Here are just some words so that we can see what happens.</code>
<code>to 4cm</code>	<code>Here are just some words so that we can see what happens.</code>
<code>to \hsize</code>	<code>Here are just some words so that we can see what happens.</code>
<code>spread 1cm</code>	<code>Here are just some words so that we can see what happens.</code>
<code>spread -1cm</code>	<code>Here are just some words so that we can see what happens.</code>
<code>adapt -1000</code>	<code>Here are just some words so that we can see what happens.</code>
<code>adapt -750</code>	<code>Here are just some words so that we can see what happens.</code>
<code>adapt -500</code>	<code>Here are just some words so that we can see what happens.</code>
<code>adapt 0</code>	<code>Here are just some words so that we can see what happens.</code>
<code>adapt 500</code>	<code>Here are just some words so that we can see what happens.</code>
<code>adapt 750</code>	<code>Here are just some words so that we can see what happens.</code>
<code>adapt 1000</code>	<code>Here are just some words so that we can see what happens.</code>

When a box is typeset the natural glue width is used but when the required width exceeds the natural width the glue stretch components kick in. With a negative spread the shrink is used but you can get underflows. The `adapt` feature freezes the glue and it removes the stretch and shrink after applying it to the glue width with the given scale factor. So, in order to get the minimum width you use `adapt -1000`.

The reason why I decided to add this feature is that when experimenting with math alignments I wanted to be able to see what shrink could be achieved.¹⁸ The next example shows this:

	
<code>to 4cm</code>	
<code>to \hsize</code>	
<code>spread 1cm</code>	
<code>spread -1cm</code>	
<code>adapt -1000</code>	
<code>adapt -750</code>	
<code>adapt -500</code>	

¹⁶ I actually did prototype it in Lua first but wanted a more natural integration in the end.

¹⁷ For the moment this keyword only has effect for horizontal boxes.

¹⁸ At that time Mikael and I were experimenting with consistent spacing in math alignments.

`adapt 0` 
`adapt 500` 
`adapt 750` 
`adapt 1000` 

Once we had this new feature it made sense to add support for it to `\framed`, one of the oldest macros that got extended over time:

```

\inframed[adaptive=1000] {Just some words}
\inframed[adaptive=500]  {Just some words}
\inframed[adaptive=0]    {Just some words}
\inframed[adaptive=-500] {Just some words}
\inframed[adaptive=-1000]{Just some words}

```

This renders as:

Just some words

Once we have it there other mechanisms can benefit from it, for instance natural tables. But keep in mind that spaces are fixed in there so there is only the expected result if glue has stretch or shrink.

10 The curious case of `\over`

Normally \TeX scans forward but there are a few special cases. First of all, \TeX is either scanning regular content or it is scanning alignments. That results in intercepts in all kind of places. When a row ends, scanning for inter-row actions happens. When the preamble is scanned there is some lookahead with partial expansion. This has side effects but these can be avoided in $\text{LuaMeta}\TeX$ by several options. Another special case is math mode. Normally curly braces indicate grouping but not in math mode: there they construct an atom, ordinary by default. Although most math constructs actually pick up some following atom, in which case we get a wrapped construct that actually is processed in a nested call to the math processing routine. That whole has a class and although in $\text{LuaMeta}\TeX$ we can make atoms with a different left end right class, normally what is inside is hidden stays hidden.¹⁹

Fraction commands like `\over` and `\above` are used like this:

```
a + 1 \over 2 + b
```

and as there can be more than for instance single digits we can do:

```
a + {12} \over {34} + b
```

but it doesn't end there because you actually need to wrap:

```
a + {{12} \over {34}} + b
```

If you don't do this will become part of the fraction. The curly braces here make the `{}`, `{}` and the whole fraction made from them ordinary atoms. Because that also influences spacing one should be aware of side effects.

```
a + {{12} \over {34}} + b
```

The argument of `\simplicity` of input is easily defeated by using

```
a + \frac{12}{34} + b
```

because it also reads sequential, is in sync with other commands like `\sqrt` and actually uses less tokens. It used more runtime, also because $\text{Con}\TeX$ t adds plenty of control and extras but you won't notice it.

Because in $\text{Con}\TeX$ t we assume users to use `\frac` it made sense to see if we can make curly braces act like groups. In $\text{LuaMeta}\TeX$ we already have `\beginmathgroup` and `\endmathgroup` that provide grouping with `\mathstyle` recovery, and by setting `\mathgroupingmode` to a non-zero value curly braces will act like these.

The effects are subtle:

```
$ a + { \bf x } ^ 2 + { \bf 1 } + { \red 123 } +  
\frac{1}{2} + {\scriptstyle 123} + \dd $
```

¹⁹ We can think of optionally exposing the edge classes but although it is easy to implement we see no reason to do that now. After all, the information is actually available already via variables.

|ordbin |binord |ordbin |binord|ordbin |binord |ordbin |binfr|arabin |binord |ordbin |bindif

`\mathgroupingmode 0`

|ordbin |binord |ordbin |bindig|digbin |bindig |digbin |binfr|arabin |bindig |digbin |bindif

`\mathgroupingmode 1`

If you see the differences you might be happy with this new trick, if not, you probably are not that much into optimal math spacing anyway and you can forget about what you just read.

11 Getting rid of jit

At the ntg meeting there was a short discussion about performance of the OpenType font machinery. Currently we still support LuaJiT_{TEX} and although the MkIV code base is mostly separated from the LMTX one there is still some compromise going on, as some Lua code is shared and needs to adapt to the fact that LuaJIT is stuck to Lua5.2 (sort of). One reason why we still support LuaJiT_{TEX} is that there are users who like the performance gain. However, if these can switch to ConT_{EX}t we could get rid of LuaJiT_{TEX} support. After all, LuaJIT is stalled as is ffi. Of course a plain T_{EX} users can object to using ConT_{EX}t but one can just use the basics and be as plain as possible.

Performance of LuaMetaT_{EX} is quite okay and it often performs better than LuaT_{EX} or even LuaJiT_{TEX}. One border case is for instance the somewhat overdone in terms of split feature steps is the Brill font. So, I decided to do some tests on my 2017 laptop that had replaced the 2013 one (Windows 10). We use cross compiled binaries. Here is the test:

```
% \enableexperiments[fonts.compact]

\definefont[Fa][brill*default @ 10pt]
\definefont[Fb][brill*default @ 12pt]
\definefont[Fc][brill-bf*default @ 10pt]
\definefont[Fd][brill-bf*default @ 12pt]

\start \testfeatureonce{1000}{
  \Fa \samplefile{tufte} \samplefile{tufte}\par
}\stop \page \edef\TimeA{\elapsedtime}
\start \testfeatureonce{1000}{
  \Fa \samplefile{tufte}\par\Fb\samplefile{tufte}\par
}\stop \page \edef\TimeB{\elapsedtime}
\start \testfeatureonce{1000}{
  \Fa \samplefile{tufte} \Fb\samplefile{tufte}\par
}\stop \page \edef\TimeC{\elapsedtime}
\start \testfeatureonce{1000}{
  \Fa \samplefile{tufte} \Fd\samplefile{tufte}\par
  \Fb \samplefile{tufte} \Fc\samplefile{tufte}\par
}\stop \page \edef\TimeD{\elapsedtime}

\startTEXpage[offset=10pt]
  \strut\infofont
  2 par 1 font : \TimeA\par
  2 par 2 font : \TimeB\par
  1 par 2 font : \TimeC\par
  1 par 4 font : \TimeD\par
\stopTEXpage
```

The next table shows the best times from three tests where each one produces 1693 pages in the default layout. We're talking of one run as normally ConT_{EX}t will run till a two pass stable state is reached (unless it is forced to one run), although in practice fixing a few typos will not for an extra run. Keep in mind that in LuaMetaT_{EX} we have a Lua driven backend that is more flexible with respect to fonts but therefore also adds some extra overhead. Also keep in mind that four different fonts per paragraph is a rare case.

	2 pars 1 font	2 pars 2 fonts	1 par 2 fonts	1 par 4 fonts
luametateX	8.9	9.4	12.1	24.6
luametateX compact	8.9	9.3	9.3	23.9
luatex	10.0	10.3	14.5	31.2
luajitteX	8.0	8.1	11.4	23.2

One should take these measurements with a grain of salt because it also depends on the system load, but it shows that there is no real need to favor a LuaJiT_EX setup over a LuaMetaT_EX one. In the meantime the default LuaT_EX binaries exceed 7 MB (and the hb variant adds quite a bit more) which LuaMetaT_EX stays around 3 MB which is nice for high performance setups with thousands of (small) runs.

Just for the record, when we use Dejavu Serif we get 2767 pages and the following timings. Again, the differences between LuaMetaT_EX and LuaJiT_EX is not that significant, especially when you realize that we're not doing anything fancy that more runtime. In practice fonts are only part of the story.

	2 pars 1 font	2 pars 2 fonts	1 par 2 fonts	1 par 4 fonts
luametateX	4.2	4.4	5.0	10.8
luametateX compact	4.2	4.5	4.5	10.5
luatex	4.9	5.0	6.2	13.3
luajitteX	4.0	4.0	5.0	10.3

12 Issues in math fonts

12.1 Introduction

After trying to improve math rendering of OpenType math fonts, we²⁰ ended up with a mix of improving the engine and fixing fonts runtime, and we are rather satisfied with the results so far.

However, as we progress and also improve the more structural and input related features of ConT_EXt, we wonder why we don't simply are more drastic when it comes to fonts. The OpenType specifications are vague, and most existing OpenType math fonts use a mixture of the OpenType features and the old T_EX habits, so we are sort of on our own. The advantage of this situation is that we feel free to experiment and do as we like.

In another article we discuss our issues with Unicode math, and we have realized that good working solutions will be bound to a macro package anyway. Also, math typesetting has not evolved much after Don Knuth set the standard, even if the limitations of those times in terms of memory, processing speed and font technologies have been lifted already for a while. And right from the start Don invited users to extend and adapt T_EX to one's needs.

Here we will zoom in on a few aspects: font parameters, glyph dimensions and properties and kerning of scripts and atoms. We discuss OpenType math fonts only, and start with a summary of how we tweak them. We leave a detailed engine discussion to a future article, since that would demand way more pages, and could confuse the reader.

12.2 Tweaks, also known as goodies

The easiest tweaks to describe are those that wipe features. Because the T_EX Gyre fonts have many bad top accent anchors (they sit above the highest point of the shape) the `wipeanchors` tweak can remove them, and we do that per specified alphabet.

7̂

In a similar fashion we `wipeitalics` from upright shapes. Okay, maybe they can play a role for subscript placement, but then they can also interfere, and they do not fit with the OpenType specification. The `wipecues` tweak zeros the dimensions of the invisible times and friends so that they don't interfere and `wipevariants` gets rid of bad variants of specified characters.

The fixers is another category, and the names indicate what gets fixed. Tweaks like these take lists of code points and specific properties to fix. We could leave it to your imagination what `fixaccents`, `fixanchors`, `fixellipses`, `fixoldschool`, `fixprimes`, `fixradicals` and `fixslashes` do, but here are some details. Inconsistencies in the dimensions of accents make them jump all over the place so we normalize them. We support horizontal stretching at the engine level.

|ordbin |binord|ordbin |binord|ordbin |binord|accrcl |relacc |ordbin |binord|ordbin |binord |ordbin |binord|ordbin |binord

It required only a few lines of code thanks to already present scaling features.

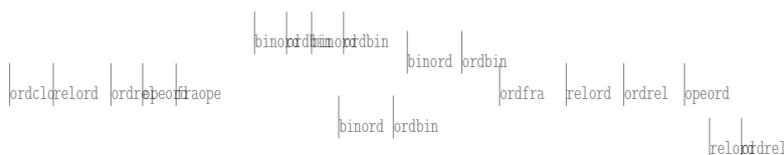
²⁰ Mikael Sundqvist and Hans Hagen

Anchors can be off so we fix these in a way so that they look better on especially italic shapes. We make sure that the automated sizing works consistently, as this is driven by width and overshoot. Several kind of ellipses can be inconsistent with each other as well as with periods (shape and size wise) so we have to deal with that. Radicals and other extensibles have old school dimensions (T_EX fonts have a limited set of widths and heights). We need to fix for instance fences of various size because we want to apply kerns to scripts on the four possible corners for which we need to know the real height and depth,

Discussing primes would take many paragraphs so we stick to mentioning that they are a mess. We now have native prime support in the engine as well as assume properly dimensioned symbols to be used. Slashes are used for skewed fractions so we'd better make sure they are set up right.

A nice tweak is **replacealphabets**. We use this to provide alternative script (roundhand) and calligraphic (chancery) alphabets (yes we have both natively in ConT_EXt while Unicode combines them in one alphabet). Many available OpenType math fonts come with one of the two alphabets only, some with roundhand and some with chancery. For the record: this tweak replaces the older **variants** tweak that filtered scripts from a stylistic font feature.

We also use the **replacealphabets** tweak to drop in Arabic shapes so that we can do bidirectional math. In practice that doesn't really boil down to a replacement but more to an addition. The **addmirrors** features accompanies this, and it is again a rather small extension to the engine to make sure we can do this efficiently: when a character is looked up we check a mirror variant when we are in r2l mode, just like we look up a smaller variant when we're in compact font mode (a ConT_EXt feature).



Another application of **replacealphabets** is to drop in single characters from another font. We use this for instance to replace the ‘not really an alpha’ in Bonum by one of our own liking. Below we show a math italic a and the original alpha, together with the modified alpha.



For that we ship a companion font. On our disks (and in the distribution) you can find:

```
/tex/texmf-fonts/fonts/data/cms/companion/RalphSmithsFormalScript-Companion.otf
/tex/texmf-fonts/fonts/data/cms/companion/TeXGyreBonumMath-Companion.otf
/tex/texmf-fonts/fonts/data/cms/companion/XITSMath-Companion.otf
```

All these are efficient drop-ins that are injected by the **replacealphabets**, some under user control, some always. We tried to limit the overhead and actually bidirectional math could be simplified which also had the benefit that when one does tens of thousands of bodyfont switches a bit of runtime is gained.

There are more addition tweaks: **addactuarial** creates the relevant symbols which is actually a right sided radical (the engine has support for two-sided radicals). It takes a bit of juggling with virtual glyphs and extensible recipes, but the results are rewarding.



In a similar fashion we try to add missing extensible arrows with **addarrows**, bars with **addbars**, equals with **addequals** and again using the radical mechanism fourier notation symbols (like hats) with **addfourier**. That one involves subtle kerning because these symbols end up at the right top of a fence like symbol.

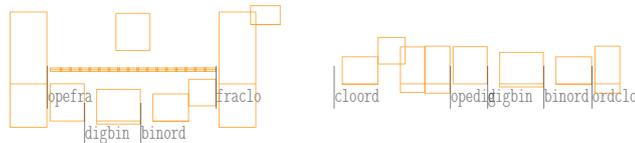
ordbin|binord|ordbin|binord |opeor|ordd|iborel |relrac|opeor|ordbin|binord|ordbin|binord |ordclo |opeor|ordclo

It was actually one of the reasons to introduce a more advanced kerning mechanism in the engine, which is not entirely trivial because one has to carry around more information, since all this is font and character bound, and when wrapped in boxes that gets hard to analyze. The **addrules** makes sure that we can do bars over and under constructs properly. The **addparts** is there to add extensible recipes to characters.

Some of these tweaks actually are not new and are also available in MkIV but more as features (optionally driven by the goodie file). An example is **addscripts** that is there for specially positioned and scaled signs (high minus and such) but that tweak will probably be redone as part of the “deal with all these plus and minus issues”. The dedicated to Alan Braslau **addprivates** tweak is an example of this: we add specific variants for unary minus and plus that users can enable on demand, which in turn of course gives class specific spacing, but we promised not to discuss those engine features here.

opepeor|ordbin |bindig|digclo|clobin |binopeor|ordbin |bindig|digclo |cloc|modif unafra

There is a handful of tweaks that deals with fixing glyph properties (in detail). We mention: **dimensions** and **accentdimensions** that can reposition in the boundingbox, fix the width and italic correction, squeeze and expand etc. The **kernpairs** tweak adds kern pairs to combinations of characters. The **kerns** provides a way to add top left, bottom left, top right and bottom right kerns and those really make the results look better so we love it!



The **margins** tweak sets margin fields that the engine can use to calculate accents over the base character better. The same is true for **setovershoots** that can make accents lean over a bit. The **staircase** feature can be used to add the somewhat complicated OpenType kerns. From all this you can deduce that the engine has all types of kerning that OpenType requires, and more.

Accents as specified in fonts can be a pain to deal with so we have more tweaks for them: **copyaccents** moves them to the right slots and **extendaccents** makes sure that we can extend them. Not all font makers have the same ideas about where these symbols should sit and what their dimensions should be.

The **checkspacing** tweak fixes bad or missing spacing related to Unicode character entries in the font, because after all, we might need them. We need to keep for instance MathML in mind, which means: processing content that we don't see and that can contain whatever an editor puts in. The **replacements** feature replaces one character by another from the same font. The **substitutes** replaces a character by one from a stylistic feature.

Relatively late we added the **setoptions** which was needed to control the engine for specific fonts. The rendering is controlled by a bunch of options (think of kerning, italic correction, and such). Some are per font, many per class. Because we can (and do) use mixed math fonts in a document, we might need to adapt the engine level options per font, and that is what this tweak does: it passes options to the font so that the engine can consult them and prefer them over the ‘global’ ones. We needed this for some fonts that have old school dimensions for extensibles (like Lucida), simply because they imitated Computer Modern. Normally that goes unnoticed, but, as mentioned before, it interferes with our optional kerning. The **fixoldschool**

tweak sort of can fix that too so `setoptions` is seldom needed. Luckily, some font providers are willing to fix their fonts!

We set and configure all these tweaks in a so-called goodie file, basically a runtime module that returns a Lua table with specifications. In addition to the tweaks subtable in the math namespace, there is a subtable that overloads the font parameters: the ones that OpenType specifies, but also new ones that we added. In the next section we elaborate more on these font bound parameters.

12.3 Font parameters

At some point in the upgrading of the math machinery we discussed some of the inconsistencies between the math constants of the XITS and STIX fonts. Now, one has to keep in mind that XITS was based on a first release of STIX that only had Type1 fonts so what follows should not to be seen as criticism, but more as observations and reason for discussion, as well as a basis for decisions to be made.

One thing we have to mention in advance, is that we often wonder why some weird and/or confusing stuff in math fonts go unnoticed. We have some suggestions:

- The user doesn't care that much how math comes out. This can easily be observed when you run into documents on the internet or posts on forums. And publishers don't always seem to care either. Consistency with old documents sometimes seems to be more important than quality.
- The user switches to another math font when the current one doesn't handle its intended math domain well. We have seen that happening and it's the easiest way out when you have not much control anyway (for instance when using online tools).
- The user eventually adds some skips and kerns to get things right, because after all \TeX is also about tweaking.
- The user doesn't typeset that complex math. It's mostly inline math with an occasional alignment (also in text style) and very few multi-level display math (with left and right fences that span at most a fraction).

We do not claim to be perfect, but we care for details, so let's go on. The next table shows the math constants as they can be found in the Stix (two) and Xits (one) fonts. When you typeset with these fonts you will notice that Xits is somewhat smaller, so two additional columns show the values compensated for the axis height and accent base height.

constant	stix	xits	base	axis	relevance
AccentBaseHeight	480	450	480	464	optional**
AxisHeight	258	250	267	258	mandate
DelimitedSubFormulaMinHeight	1325	1500	1600	1548	
DisplayOperatorMinHeight	1800	1450	1547	1496	
FlattenedAccentBaseHeight	656	662	706	683	optional**
FractionDenominatorDisplayStyleGapMin	150	198	211	204	
FractionDenominatorDisplayStyleShiftDown	640	700	747	722	
FractionDenominatorGapMin	68	66	70	68	
FractionDenominatorShiftDown	585	480	512	495	
FractionNumeratorDisplayStyleGapMin	150	198	211	204	
FractionNumeratorDisplayStyleShiftUp	640	580	619	599	
FractionNumeratorGapMin	68	66	70	68	
FractionNumeratorShiftUp	585	480	512	495	

FractionRuleThickness	68	66	70	68	optional
LowerLimitBaselineDropMin	670	600	640	619	
LowerLimitGapMin	135	150	160	155	
MathLeading	150	150	160	155	
MinConnectorOverlap	100	50	53	52	mandate
OverbarExtraAscender	68	66	70	68	
OverbarRuleThickness	68	66	70	68	optional*
OverbarVerticalGap	175	198	211	204	
RadicalDegreeBottomRaisePercent	55	70	75	72	mandate
RadicalDisplayStyleVerticalGap	170	186	198	192	
RadicalExtraAscender	78	66	70	68	
RadicalKernAfterDegree	-335	-555	-592	-573	
RadicalKernBeforeDegree	65	277	295	286	
RadicalRuleThickness	68	66	70	68	
RadicalVerticalGap	85	82	87	85	
ScriptPercentScaleDown	70	75	80	77	
ScriptScriptPercentScaleDown	55	60	64	62	
SkewedFractionHorizontalGap	350	300	320	310	
SkewedFractionVerticalGap	68	66	70	68	
SpaceAfterScript	40	41	44	42	
StackBottomDisplayStyleShiftDown	690	900	960	929	
StackBottomShiftDown	385	800	853	826	
StackDisplayStyleGapMin	300	462	493	477	
StackGapMin	150	198	211	204	
StackTopDisplayStyleShiftUp	780	580	619	599	
StackTopShiftUp	470	480	512	495	
StretchStackBottomShiftDown	590	600	640	619	
StretchStackGapAboveMin	68	150	160	155	
StretchStackGapBelowMin	68	150	160	155	
StretchStackTopShiftUp	800	300	320	310	
SubSuperscriptGapMin	150	264	282	272	
SubscriptBaselineDropMin	160	50	53	52	
SubscriptShiftDown	210	250	267	258	
SubscriptTopMax	368	400	427	413	
SuperscriptBaselineDropMax	230	375	400	387	
SuperscriptBottomMaxWithSubscript	380	400	427	413	
SuperscriptBottomMin	120	125	133	129	
SuperscriptShiftUp	360	400	427	413	
SuperscriptShiftUpCramped	252	275	293	284	
UnderbarExtraDescender	68	66	70	68	
UnderbarRuleThickness	68	66	70	68	optional*
UnderbarVerticalGap	175	198	211	204	
UpperLimitBaselineRiseMin	300	300	320	310	
UpperLimitGapMin	135	150	160	155	

Very few values are the same. So, what exactly do these constants tell us? You can even wonder why they are there at all. Just think of this: we want to typeset math, and we have an engine that we can control. We know how we want it to look. So, what do these constants actually contribute? Plenty relates to the height and depth of the nucleus and/or the axis. The fact that we have to fix some in the goodie files, and the fact

that we actually need more variables that control positioning, makes for a good argument to just ignore most of the ones provided by the font, especially when they seem somewhat arbitrarily. Can it be that font designers are just gambling a bit, looking at another font, and starting from there?

The relationship between \TeX 's math font parameters and the OpenType math constants is not one-to-one. Mapping them onto each other is possible but actually is font dependent. However, we can assume that the values of Computer Modern are leading.

The `AxisHeight`, `AccentBaseHeight` and `FlattenedAccentBaseHeight` are set to the x-height, a value that is defined in all fonts. The `SkewedFractionVerticalGap` also gets that value. Other variables relate to the em-width (or `\quad`), for instance the `SkewedFractionHorizontalGap` that gets half that value. Of course these last two then assume that the engine handles skewed fractions.

Variables that directly map onto each other are `StretchStackGapBelowMin` as `bigopspacing1`, `StretchStackTopShiftUp` as `bigopspacing3`, `StretchStackGapAboveMin` as `bigopspacing2` and `StretchStackBottomShiftDown` as `bigopspacing4`. However, these clash with `UpperLimitBaselineRiseMin` as `bigopspacing3`, `UpperLimitGapMin` as `bigopspacing1`, `LowerLimitBaselineDropMin` as `bigopspacing4` and `LowerLimitGapMin` as `bigopspacing2`. Where in traditional fonts these are the same, in OpenType they can be different. Should they be?

Internally we use different names for variables, simply because the engine has some parameters that OpenType maths hasn't. So we have `limit_above_kern` and `limit_below_kern` for `bigopspacing5`.

A couple of parameters have different values for (cramped) `displaystyle`. The `FractionDelimiterSize` and `FractionDelimiterDisplayStyleSize` use `delim2` and `delim1`. The `FractionDenominatorShiftDown` and `FractionDenominatorDisplayStyleShiftDown` map onto `denom2` and `denom1` and their numerator counterparts from `num2` and `num1`. The `Stack*` parameters also use these. The `sub1`, `sub2`, `sup1`, `sup2`, `sup3`, and `supdrop` can populate the `Sub*` and `Super*` parameters, also in different styles.

The rest of the parameters can be defined in terms of the default rulethickness, quad or xheight, often multiplied by a factor. For some we see the `1/18` show up a number that we also see with muskips. Some constants can be set from registers, like `SpaceAfterScript` which is just `\scriptspace`.

If you look at the Lua \TeX source you will find a section where this mapping is done in the case of a traditional font, that is: one without a math constants table. In LuaMeta \TeX we don't need to do this because font loading happens in Lua. So we simply issue an error when the math engine can't resolve a mandate parameter. The fact that we have a partial mapping from math constants onto traditional parameters and that Lua \TeX has to deal with the traditional ones too make for a somewhat confusing landscape. When in LuaMeta \TeX we assume wide fonts to be used that have a math constants table, we can probably clean up some of this.

We need to keep in mind that Cambria was the starting point, and it did borrow some concepts from \TeX . But \TeX had parameters because there was not enough information in the glyphs! Also, Cambria was meant for MS Word, and a word processor is unlikely to provide the level of control that \TeX offers, so it needs some directions with respect to e.g. spacing. Without user control, it has to come up with acceptable compromises. So actually the LuaMeta \TeX math engine can be made a bit cleaner when we just get rid of these parameters.

So, which constants are actually essential? The `AxisHeight` is important and also design related. Quite likely this is where the minus sits above the baseline. It is used for displacements of the baseline so that for instance fractions nicely align. When testing script anchored to fences we noticed that the parenthesis in XITS had too little depth while STIX had the expected amount. This relates to anchoring relative to the math axis.

Is there a reason why `UnderbarRuleThickness` and `OverbarRuleThickness` should differ? If not, then we only need a variable that somehow tells us what thickness fits best with the other top and bottom accents. It is quite likely the same as the `RadicalRuleThickness`, which is needed to extend the radical symbol. So, here three constants can be replaced by one design related one. The `FractionRuleThickness` can also be derived from that, but more likely is that it is a quantity that the macro package sets up anyway, maybe related to rules used elsewhere.

The `MinConnectorOverlap` and `RadicalDegreeBottomRaisePercent` also are related to the design although one could abuse the top accent anchor for the second one. So they are important. However, given the small number of extensibles, they could have been part of the extensible recipes.

The `AccentBaseHeight` and `FlattenedAccentBaseHeight` might relate to the margin that the designer put below the accent as part of the glyph, so it is kind of a design related constant. Nevertheless, we fix quite some accents in the goodie files because they can be inconsistent. That makes these constants somewhat dubious too. If we have to check a font, we can just as well set up constants that we need in the goodie file. Also, isn't it weird that there are no bottom variants?

We can forget about `MathLeading` as it serves no purpose in \TeX . The `DisplayOperatorMinHeight` is often set wrong so although we fix that in the goodie file it might be that we just can use an internal variable. It is not the font designer who decides that anyway. The same is true for `DelimitedSubFormulaMinHeight`.

If we handle skewed fractions, `SkewedFractionHorizontalGap` and `SkewedFractionVerticalGap` might give an indication of the tilt but why do we need two? It is design related though, so they have some importance, when set right.

The rest can be grouped, and basically we can replace them by a consistent set of engine parameters. We can still set them up per font, but at least we can then use a clean set. Currently, we already have more. For instance, why only `SpaceAfterScript` and not one for before, and how about prescripts and primes? If we have to complement them with additional ones and also fix them, we can as well set up all these script related variables.

For fractions the font provides `FractionDenominatorDisplayStyleGapMin`, `FractionDenominatorDisplayStyleShiftDown`, `FractionDenominatorGapMin`, `FractionDenominatorShiftDown`, `FractionNumeratorDisplayStyleGapMin`, `FractionNumeratorDisplayStyleShiftUp`, `FractionNumeratorGapMin` and `FractionNumeratorShiftUp`. We might try to come up with a simpler model.

Limits have: `LowerLimitBaselineDropMin`, `LowerLimitGapMin`, `UpperLimitBaselineRiseMin` and `UpperLimitGapMin`. Limits are tricky anyway as they also depend on abusing the italic correction for anchoring.

Horizontal bars are driven by `OverbarExtraAscender`, `OverbarVerticalGap`, `UnderbarExtraDescender` and `UnderbarVerticalGap`, but for e.g. arrows we are on our own, so again a not so useful set.

Then radicals: we need some more than these `RadicalDisplayStyleVerticalGap`, `RadicalExtraAscender`, `RadicalKernAfterDegree`, `RadicalKernBeforeDegree` and `RadicalVerticalGap`, and because we really need to check these there is no gain having them in the font.

Isn't it more a decision by the macro package how script and scriptscript should be scaled? Currently we listen to `ScriptPercentScaleDown` and `ScriptScriptPercentScaleDown`, but maybe it relates more to usage.

We need more control than just `SpaceAfterScript` and an engine could provide it more consistently. It's a loner.

How about `StackBottomDisplayStyleShiftDown`, `StackBottomShiftDown`, `StackDisplayStyleGapMin`, `StackGapMin`, `StackTopDisplayStyleShiftUp` and `StackTopShiftUp`? And isn't this more for the renderer to decide: `StretchStackBottomShiftDown`, `StretchStackGapAboveMin`, `StretchStackGapBelowMin` and `StretchStackTopShiftUp`?

This messy bit can also be handled more convenient so what exactly is the relationship with the font design of `SubSuperscriptGapMin`, `SubscriptBaselineDropMin`, `SubscriptShiftDown`, `SubscriptTopMax`, `SuperscriptBaselineDropMax`, `SuperscriptBottomMaxWithSubscript`, `SuperscriptBottomMin`, `SuperscriptShiftUp` and `SuperscriptShiftUpCramped`?

Just for the record, here are the (font related) ones we added so far. A set of prime related constants similar to the script ones: `PrimeRaisePercent`, `PrimeRaiseComposedPercent`, `PrimeShiftUp`, `PrimeBaselineDropMax`, `PrimeShiftUpCramped`, `PrimeSpaceAfter` and `PrimeWidthPercent`. Of course, we also added `SpaceBeforeScript` just because we want to be symmetrical in the engine where we also have to deal with prescripts.

These we provide for some further limit positioning: `NoLimitSupFactor` and `NoLimitSubFactor`; these for delimiters: `DelimiterPercent` and `DelimiterShortfall`; and these for radicals in order to compensate for sloping shapes: `RadicalKernAfterExtensible` and `RadicalKernBeforeExtensible` because we have doublesided radicals.

Finally, there are quite some (horrible) accent tuning parameters: `AccentTopShiftUp`, `AccentBottomShiftDown`, `FlattenedAccentTopShiftUp`, `FlattenedAccentBottomShiftDown`, `AccentBaseDepth`, `AccentFlattenedBaseDepth`, `AccentTopOvershoot`, `AccentBottomOvershoot`, `AccentSuperscriptDrop`, `AccentSuperscriptPercent` and `AccentExtendMargin`, but we tend to move some of that to the tweaks on a per accent basis.

Setting these parameters right is not trivial, and also a bit subjective. We might, however, assume that for instance the math axis is set right, but alas, when we were fixing the less and greater symbols in Lucida Bright Math, we found that all symbols actually were designed for a math axis of 325, instead of the given value 313, and that difference can be seen!



The assumption is that the axis goes through the middle of the minus. Luckily it was relatively easy to fix these two symbols (they also had to be scaled, maybe they originate in the text font?) and adapt the axis. We still need to check all the other fonts, but it looks like they are okay, which is good because the math axis plays an important role in rendering math. It is one of the few parameters that has to be present and right. A nice side effect of this is that we end up with discussing new (ConTeXt) features. One can for instance shift all non-character symbols down just a little and lower the math axis, to get a bit more tolerance in lines with many inline fractions, radicals or superscripts, that otherwise would result in interline skips.

A first step in getting out of this mess is to define *all* these parameters in the goodie file where we fix them anyway. That way we are at least not dependent on changes in the font. We are not a word processor so we have way more freedom to control matters. And preset font parameters sometimes do more harm than good. A side effect of a cleanup can be that we get rid of the evolved mix of uppercase and lowercase math control variables and can be more consistent. Ever since LuaTeX got support for OpenType, math constants names have been mapped and matched to traditional TeX font parameters.

12.4 Metrics

With metrics we refer to the dimensions and other properties of math glyphs. The origin of digital math fonts is definitely Computer Modern and thereby the storage of properties is bound to the tfm file format. That format is binary and can be loaded fast. It can also be stored in the format, unless you're using Lua_T_EX or LuaMeta_T_EX where Lua is the storage format. A tfm file stores per character the width, height, depth and italic correction. The file also contains font parameters. In math fonts there are extensible recipes and there is information about next in size glyphs. The file has kerning and ligature tables too.

Given the times _T_EX evolved in, the format is rather compact. For instance, the height, depth and italic correction are shared and indices to three shared values are used. There can be 16 heights and depths and 64 italic corrections. That way much fits into a memory word.

The documentation tells us that “The italic correction of a character has two different uses. (a) In ordinary text, the italic correction is added to the width only if the _T_EX user specifies ‘\’ after the character. (b) In math formulas, the italic correction is always added to the width, except with respect to the positioning of subscripts.” It is this last phenomena that gives us some trouble with fonts in OpenType math. The fact that traditional fonts cheat with the width and that we add and selectively remove or ignore the correction makes for fuzzy code in Lua_T_EX although splitting the code paths and providing options to control all this helps a bit. In LuaMeta_T_EX we have more control but also expect an OpenType font. In OpenType math there are italic corrections, and we even have the peculiar usage of it in positioning limits. However, the idea was that staircase kerns do the detailed relative positioning.

Before we dive into this a bit more, it is worth mentioning that Don Knuth paid a lot of attention to details. The italic alphabet in math uses the same shapes as the text italic but metrics are different as is shown below. We have also met fonts where it looked like the text italics were taken, and where the metrics were handled via more excessive italic correction, sometimes combined with staircase kerns that basically were corrections for the side bearing. This is why we always come back to Latin Modern and Cambria when we investigate fonts: one is based on the traditional _T_EX model, with carefully chosen italic corrections, and the other is based on the OpenType model with staircase kerning. They are our reference fonts.

A sample of the Latin Modern Roman Italic font. The lowercase letters 'a' through 'z' are displayed in a single line. Each letter is enclosed in a thin orange rectangular box that highlights its bounding box. The font is a classic serif italic style.

Latin Modern Roman Italic

A sample of the Latin Modern Math Italic font. The lowercase letters 'a' through 'z' are displayed in a single line. Each letter is enclosed in a thin orange rectangular box that highlights its bounding box. The font is a serif italic style, similar to the Roman Italic but with different metrics.

Latin Modern Math Italic

In Con_T_EXt MkIV we played a lot with italic correction in math and there were ways to enforce, ignore, selectively apply it, etc. But, because fonts actually demand a mixture, in LuaMeta_T_EX we ended up with more extensive runtime patching of them. Another reason for this was that math fonts can have weird properties. It looks like when these standards are set and fonts are made, the font makers can do as they like as long as the average formula comes out right, and metrics to some extent resemble a traditional font. However, when testing how well a font behaves in a real situation there can be all kind of interferences from the macro package: inter-atom kerning, spacing corrections macros, specific handling of cases, etc. We even see OpenType fonts that seem to have the same limited number of heights, depths and italic corrections. And, as a consequence we get for instance larger sizes of fences having the same depth for all the size variants, something that is pretty odd for an OpenType font with no limitations.

The italic correction in traditional \TeX math gets added to the width. When a subscript is attached to a kernel character it sits tight against that character: its position is driven by the width of the kernel. A superscript on the other hand is moved over the italic width so that it doesn't overlap or touch the likely sticking out bit of the kernel. This means that a traditional font (and quite some OpenType math fonts are modelled after Computer Modern) have to find compromises of width and italic correction for characters where the subscript is supposed to move left (inside the bounding box of the kernel).

The OpenType specification has some vague remarks about applying italic correction between the last in a series of slanted shapes and operators, as well as positioning limits, and suggests that it relates to relative super- and subscript positioning. It doesn't mention that the correction is to be added to the width. However, the main mechanism for anchoring script are these top and bottom edge kerns. It's why in fonts that provide these, we are unlikely to find italic correction unless it is used for positioning limits.

It is for that reason that an engine can produce reasonable results for fonts that either provide italics or provide kerns for anchoring: having both on the same glyph would mean troubles. It means that we can configure the engine options to add italic correction as well as kerns, assuming distinctive usage of those features. For a font that uses both we need to make a choice (this is possible, since we can configure options per font). But that will never lead to always nicely typeset math. In fact, without tweaks many fonts will look right because in practice they use some mixture. But we are not aiming at partial success, we want all to look good.

Here is another thing to keep in mind (although now we are guessing a bit). There is a limited number of heights and depths in \TeX fonts possible (16), but four times as many italic corrections can be defined (64). Is it because Don Knuth wanted to properly position the sub- and subscripts? Adding italic correction to the width is pretty safe: shapes should not overlap. Choosing the right width for a subscript needs more work because it's more visual. In the end we have a width that is mostly driven by superscript placement! That also means that as soon as we remove the italic correction things start looking bad. In fact, because also upright math characters have italic correction the term 'italic' is a bit of a cheat: it's all about script positioning and has little to do with the slope of the shapes.

One of the reasons why for instance spacing between an italic shape and an upright one in \TeX works out okay is that in most cases they come from a different font, which can be used as criterium for keeping the correction; between a sequence of same-font characters it gets removed. However, in OpenType math there is a good chance that all comes from the same font (at least in Con \TeX t), unless one populates many families as in traditional \TeX . We have no clue how other macro packages deal with this but it might well be the case that using many families (one for each alphabet) works better in the end. The engine is really shape and alphabet agnostic, but one can actually wonder if we should add a glyph property indicating the distinctive range. It would provide engine level control over a run of glyphs (like multiplying a variable represented by a greek alpha by another variable presented by an upright b).

But glyph properties cannot really be used here because we are still dealing with characters when the engine transforms the noad list into a node list. So, when we discussed this, we started wondering how the engine could know about a specific shape (and tilt) property at all, and that brought us to pondering about an additional axis of options. We already group characters in classes, but we can also group them with properties like `tilted`, `dotless`, `bold`. When we pair atoms we can apply options, spacing and such based on the specific class pair, and we can do something similar with category pairs. It basically boils down to for instance `\mccode` that binds a character to a category. Then we add a command like `\setmathcategorization` (analogue to `\setmathspacing`) that binds options to pairs of categories. An easier variant of this might be to let the `\mccode` carry a (bit)set of options that then get added to the already existing options that can be bound to character noads as we create them. This saves us some configuration. Deciding what suits best depends on what we want to do: the fact that \TeX doesn't do this means that probably no one ever

gave it much thought, but once we do have this mechanism it might actually trigger demand, if only by starting at existing documents where characters of a different kind sit next to each other (take this ‘a’ invisible times ‘x’). It would not be the first time that (in ConT_EXt) the availability of some feature triggers creative (ab)usage.

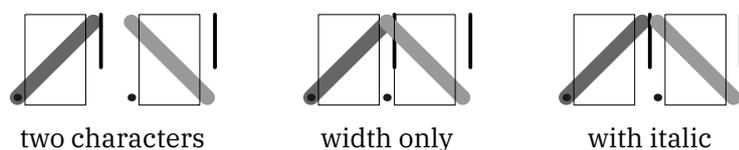
Because the landscape has settled, because we haven't seen much fundamental evolution in OpenType math, because in general T_EX math doesn't really evolve, and because ConT_EXt in the past has not been seen as suitable for math, we can, as mentioned before, basically decide what approach we follow. So, that is why we can pick up on this italic correction in a more drastic way: we can add the correction to the width, thereby creating a nicely bounded glyph, and moving the original correction to the right bottom kern, as that is something we already support. In fact, this feature is already available, we only had to add setting the right bottom kern. The good news is that we don't need to waste time on trying to get something extra in the font format, which is unlikely to happen anyway after two decades.

It is worth noticing that when we were exploring this as part of using MetaPost to analyze and visualize these aspects, we also reviewed the `wipeitalics` tweak and wondered if, in retrospect, it might be a dangerous one when applied to alphabets (for digits and blackboard bold letters it definitely makes sense): it can make traditional super- and subscript anchoring less optimal. However, for some fonts we found that improper bounding boxes can badly interfere anyway: for instance the upright ‘f’ in EBGaramond sticks out left and right, and has staircase kerns that make scripts overlap. The right top of the shape sticks out a lot and that is because the text font variant is used. We already decided to add a `moveitalics` tweak that moves italic kerns into the width and then setting a right bottom kern that compensates it that can be a pretty good starting point for our further exploration of optimal kerns at the corners. That tweak also fixes the side bearings (negative llx) and compensates left kerns (when present) accordingly. An additional `simplifykerns` tweak can later migrate staircase kerns to simple kerns.

So, does that free us from tweaks like `dimensions` and `kerns`? Not completely. But we can forget about the italic correction in most cases. We have to set up less lower right kerns and maybe correct a few. It is just a more natural solution. So how about these kerns that we need to define? After all, we also have to deal with proper top kerns, and like to add kerns that are not there simply because the mentioned comprise between width, italic and the combination was impossible. More about that in the next section.

12.5 Kerning

In the next pictures we will try to explain more visual what we have in mind and are experimenting with as we write this. In the traditional approach we have shapes that can communicate the width, height, depth and italic correction to the engine so that is what the engine can work with. The engine also has the challenge to anchor subscripts and superscripts in a visual pleasing way.

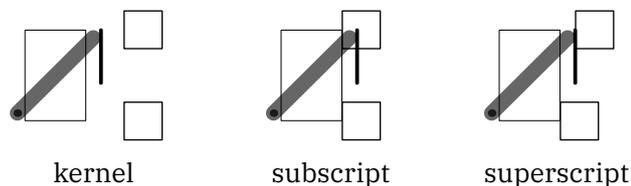


In this graphic we show two pseudo characters. The shown bounding box indicates the width as seen by the engine. An example of such a shape is the math italic f, and as it is used a lot in formulas it is also one of the most hard ones to handle when it comes to spacing: in nearly all fonts the right top sticks out and in some fonts the left part also does that. Imagine how that works out with scripts, fences and preceding characters.

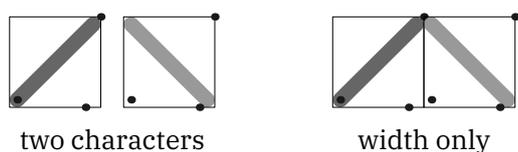
When we put two such characters together they will overlap, and this is why we need to add the italic correction. That is also why the T_EX documentation speaks in terms of “always add the italic correction to the

width". This also means that we need to remove it occasionally, something that you will notice when you study for instance the LuaTeX source, that has a mix of traditional and OpenType code paths. Actually, compensating can either be done by changing the width property of a glyph node or by explicitly adding a kern. In LuaMetaTeX we always add real kerns because we can then trace better.

The last graphic in the above set shows how we compensate the width for the bit that sticks out. It also shows that we definitely need to take neighboring shapes into account when we determine the width and italic correction, especially when the later is *not* applied (read: removed).

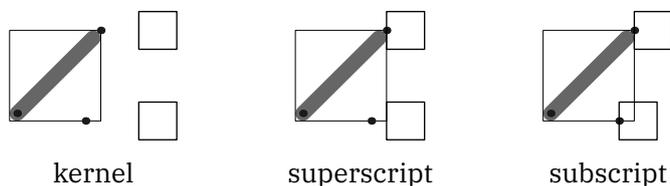


Here we anchored a super- and subscript. The subscript position it tight to the advance width, again indicated by the box. The superscript however is moved by the italic correction and in the engine additional spacing before and after can be applied as well, but we leave that for now. It will be clear that when the font designer chooses the width and italic correction, the fact that scripts get attached has to be taken into account.



In this graphic we combine the italic correction with the width. Keep in mind that in these examples we use tight values but in practice that correction can also add some extra right side bearing (white space). This addition is an operation that we can do when loading a font. At the same time we also compensate the left edge for which we can use the x coordinate of the left corner of the glyphs real bounding box. The advance width starts at zero and that corner is then left of the origin. By looking at shapes we concluded that in most cases that shift is valid for usage in math where we don't need that visual overlap. In fact, when we tested some of that we found that the results can be quite horrible when you don't do that; not all fonts have left bottom kerning implemented.

The dot at the right is actually indicating the old italic correction. Here we let it sit on the edge but as mentioned there can be additional (or maybe less) italic correction than tight.

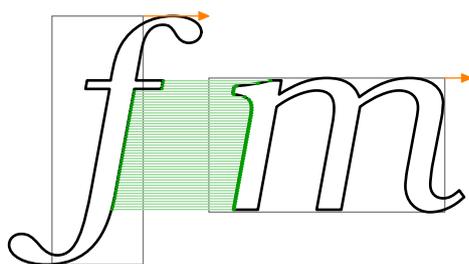


Finally we add the scripts here. This time we position the superscript and subscript at the top and bottom anchors. The bottom anchor is, as mentioned, the old italic correction, and the top one currently just the edge. And this is what our next project is about: identify the ideal anchors and use these instead.

In the ConTeXt goodie files (the files that tweak the math fonts runtime) we can actually already set these top and bottom anchors and the engine will use them when set. These kerns are not to be confused with the more complicated staircase kerns. They are much simpler and lightweight. The fact that we already have them makes it relatively easy to experiment with this.

It must be noted that we talk about three kinds of kerns: inter character kerns, corner kerns and staircase kerns. We can set them all up with tweaks but so far we only did that for the most significant ones, like integrals. The question is: can we automate this? We should be careful because the bad top accent anchors in the \TeX Gyre fonts demonstrate how flawed heuristics can be. Interesting is that the developers of these font used MetaPost and are highly qualified in that area. And for us using MetaPost is also natural!

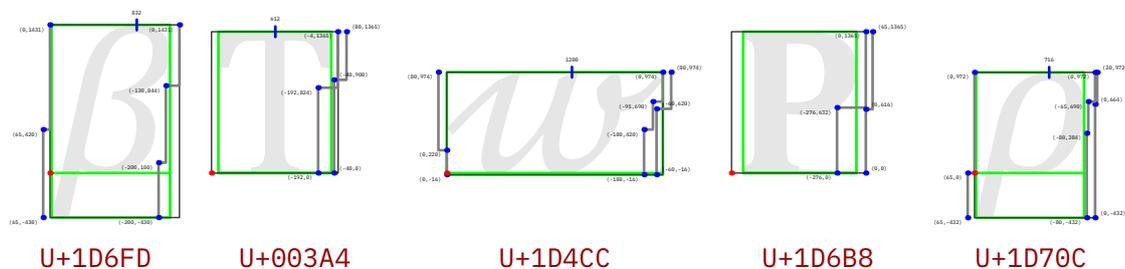
The approach that we follow is somewhat interactive. When working on the math update we like to chat (with zoom) about these matters. We discuss and explore plenty and with these kerns we do the same. Because MetaPost produces such nice and crispy graphics, and because MetaFun is well integrated into Con \TeX t we can link all these subsystems and just look at what we get. A lot is about visualization: if we discuss so called ‘grayness’ in the perspective of kerning, we end up with calculating areas, then look at what it tells us and as a next step figure out some heuristic. And of course we challenge each other into new trickery.



We are sure that getting this next stage in the perfection of math typesetting in Con \TeX t and LuaMeta \TeX will take quite some time, but the good news is that all machinery is in place. We also have to admit that it all might not work out well, so that we stick to what we have now. But at least we had the fun then. And it is also a nice example of both applying mathematics and programming graphics.

That said, if it works out well, we can populate the goodie files with output from MetaPost, tweak a little when needed, and that saves us some time. One danger is that when we try to improve rendering the whole system also evolves which in turn will give different output, but we can always implement all this as features because after all Con \TeX t is very much about configuration. And it makes nice topics for articles and talks too!

The kerns discussed in the previous paragraphs are not the ones that we find in OpenType fonts. There we have ‘staircase’ kerns that stepwise go up or down by height and kern. So, one can have different kerns depending on the height and sort of follow the shape. This permits quite precise kerning between for instance the right bottom of a kernel and left top of a subscript. So how is that used in practice? The reference font Cambria has these kerns but close inspection shows that these are not that accurate. Fortunately, we never enter the danger zone with subscripts, because other parameters prevent that. If we look at for instance Lucida and Garamond, then we see that their kerns are mostly used as side bearing, and not really as staircase kerns.



In these figures you see a few glyphs from cambria with staircase kerns and although we show them small you will notice that some kern boundaries touch the shape. As subscripts never go that high it goes unnoticed but it also shows that sticking to the lowest boundary makes sense.

We conclude that we can simplify these kerns, and just transform them into our (upto four) corner kerns. It is unlikely that Cambria gets updates and that other fonts become more advanced. One can even wonder if multiple steps really give better results. The risk of overlap increases with more granularity because not every pair of glyphs is checked. Also, the repertoire of math characters will likely not grow and include shapes that differ much from what we can look at now. Reducing these kerns to simple ones, that can easily be patched at will in a goodie file, has advantages. We can even simplify the engine.

12.6 Conclusion

So how can we summarize the above? The first conclusion is that we can only get good results when we runtime patch fonts to suite the engine and our (ConT_EXt) need. The second conclusion is that we should seriously consider to drop (read: ignore) most math font parameter and/or to reorganize them. There is no need to be conforming, because these parameters are often not that well implemented (thumb in mouth). The third conclusion (or observation) is that we should get rid of the excessive use of italic correction, and go for our new corner kerns instead. Last, we can conclude that it makes sense to explore how we can use MetaPost to analyze the shapes in such a way that we can improve inter character kerning, corner kerns and maybe even, in a limited way, staircase kerns.

And, to come back to accents: very few characters need a top kern. Most can be handled with centered anchors, and we need tweaks for margins and overshoot anyway. The same is true for many other tweaks: they are there to stay.

This is how we plan to go forward:

- We pass no italic corrections in the math fonts to the engine, but instead we have four dedicated simple corner kerns, top and bottom anchors, and we also compensate negative left side bearing. We should have gone that route earlier (as follow up on a MkIV feature) but were still in some backward compatibility mindset.
- The LuaMetaT_EX math engine might then be simplified by removing all code related to italic correction. Of course it hurts that we spent so much time on that over the years. We can anyway disable engine options related to italic correction in the ConT_EXt setup. Of course the engine is less old school generic then but that is the price of progress.
- A default goodie file is applied that takes care of this when no goodie file is provided. We could do some in the engine, but there is no real need for that. We can simplify the mid 2022 goodie files because we have to fix less glyphs.
- If we ever need italic correction (that is: backtrack) then we use the (new) `\mccode` option code that can identity sloped shapes. But, given that ignoring the correction between sloped shapes looks pretty bad, we can as well forget about this. After all, italic correction never really was about correcting italics, but more about anchoring scripts.
- Staircase kerns can be reduced to simple corner kerns and the engine can be simplified a bit more. In the end, all we need is true widths and simple corner kerns.
- We reorganize the math parameters and get rid of those that are not really font design dependent. This also removes a bit of overlap. This will be done as we document.
- Eventually we can remove tweaks that are no longer needed in the new setup, which is a good thing as it also save us some documenting and maintenance.

All this will happen in the perspective of ConT_EXt and LuaMetaT_EX but we expect that after a few years of usage we can with confidence come to some conclusions that can trickle back in the other engines so that other macro packages can benefit from a somewhat radical different but reliable approach to math rendering, one that works well with the old and new fonts.

13 Gaining performance

In the meantime (2022) the LuaMetaT_EX engine has touched many aspects of the original T_EX implementation. This has resulted in less memory consumption than for instance LuaT_EX when we talk tokens, more efficient macro handing, additional storage options and numerous new features and optimizations. Of course one can disagree about all of this, but what matters to us is that it facilitates ConT_EXt well. That macro package went from MkII to MkIV to MkXL (aka LMTX).

Although over the years the macros evolved the basic ideas haven't changed: it is a keyword driven macro package that is set up in a way that makes it possible to move forward. In spite of what one might think, the fundamentals didn't change much. It looks like we made the right decisions at the start, which means that we can change low level implementations to match the engine without users noticing much. Of course in the area of fonts, input encoding and languages things have changed simply because the environment in which we operate changes.

A fundamental difference between pdfT_EX and LuaMetaT_EX is that the later is in many aspects 32 and even 64 bit all over the place. That comes with a huge performance hit but also with possibilities (that I won't discuss here now)! On a simple document nothing can beat pdfT_EX, even with the optimizations that we can apply when using the modern engines. However, on more complex documents reality is that LuaMetaT_EX can outperform pdfT_EX, and documents (read: user demands) have become more complex indeed.

So, how does that work in practice? One can add some features to an engine but then the macro package has to be adapted. Due to the way ConT_EXt is organized it was not that hard to keep it in sync with new features, although not all are applied yet to full extend. Some new features improved performance, others made the machinery (or its usage) a bit slower. The first versions of LuaMetaT_EX were some 25% slower than LuaT_EX, simply because the backend is written in Lua. But, end 2022 we can safely say that LuaMetaT_EX can be 50% faster than its ancestor. This is due to a mix of the already mentioned optimizations and new features, for instance a more powerful macro parser. The backend has become more complex too, but also benefits from a few more helpers.

Because we spend a lot of time in Lua the interfaces to T_EX have been extended and improved too. Of course we depend on the Lua interpreter being kept in optimum state by its authors. It must be said that quite some of the interfaces might look obscure but these are not really meant for the average user anyway. Also, as soon as one messes with tokens and nodes at that level one definitely need to know what one's doing!

The more stable the engine becomes, the less there is to improve. Occasionally it was possible to squeeze our a few more milliseconds on run but it depends a lot of what one does. And T_EX is already quite fast anyway. Of course 0.005 seconds on a 5 second run is not much but hundred times such an improvement is noticeable, especially when there are multiple runs or when one processes a batch of 10.000 documents (each needing two runs).

One interesting aspect of T_EX that it can surprise you every now and then. End 2022 I decided to play a bit more with a feature that has been around for a while:

```
\integerdef \fooA 123
\dimensiondef\fooB 123pt
```

These primitives create a counter and a dimen where the value is stored in the hash table. The original reason was that I didn't want to spoil registers. But although these are basically constants there is more to it now.

```
\countdef\fooC 27
\dimendef\fooD 56
```

These primitives create a command that stores the register number (here 27 and 56) with the name. In this case a ‘variable’ is accessed in two steps: the `\fooC` macro expands to an register accessor with value 27. Next that accessor will kick in and fetch (or set) the value in slot 27 of the memory range bound to (in total 65K) counters. All these registers sit at the lower end of $\text{T}_{\text{E}}\text{X}$'s memory which is definitely not next to the meaning of `\fooC`. So we have two memory accesses to get to the number. Contrary to that once we are at `\fooA` we are also at the value. Although memory access can be fast when the relevant slots are cached in practice it can give delays, especially in a program like $\text{T}_{\text{E}}\text{X}$ where most data is spread all over the place. And imagine other processes competing for access too.

It is for that reason that I decided to replace the more or less ‘constant’ property of `\fooA` by one that also supports assignments as well as the arithmetic commands like `\advance`. This was not that hard due to the way the $\text{LuaMetaT}_{\text{E}}\text{X}$ source is organized. After that using these pseudo constants proved to be more efficient than registers, but of course I then had to adapt the source. Interestingly that should have been easy because one only needs to change the definitions of for instance `\newcount` but in practice that doesn't work because it will/can break for instance generic packages like Tikz.

So, in the end a new allocator was added and just over 1000 lines in some 120 files (with some overlap) had to be adapted to this. In addition some precautions had to be made for access from Lua because the quantities were no longer registers. But it was rewarding in the sense that the test suite now ran some 5% faster and processing the $\text{LuaMetaT}_{\text{E}}\text{X}$ manual went from 8.7 seconds on my laptop down to around 8.5, which is not bad.

Now why do we bother so much about performance? If I really want a faster run using a decent desktop is of more help. But even then there can be reasons. When Mikael and I were discussing math engine developments at some point we noticed that a run took twice as much time as a result of (supposedly idle) background tasks. Now keep in mind that $\text{T}_{\text{E}}\text{X}$ uses a single core so with plenty cores it should not be that bad. However, when the video chat program takes half of the CPU power, or when a mathematical manipulation program idles in the background taking 80 percent of a modern machine, or when a popular editor keeps all kind of plug ins busy for no reason, or when a supposedly closed browser consumes gigabytes of memory and keeps dozens of supposedly idle threads busy, it becomes clear that we should not let $\text{T}_{\text{E}}\text{X}$ put a large burden on memory access (and cache).

It can get even worse when one runs on virtual machines where the host suggests that you get 16 cores so that you can run a dozen $\text{T}_{\text{E}}\text{X}$ jobs in parallel but simple measurements show that these shared cores report a much higher ideal performance than the one you measure. So, the less demanding a $\text{ConT}_{\text{E}}\text{Xt}$ run becomes, the better: we're not so much after the .2 seconds on a 8 second run, but more after 3 seconds for that same run when using shared resources where it became 15 seconds. And this is what observations with respect to the performance of the test suite seem to indicate.

In the end it's mostly about comfort: when you process a document of 300 pages, 10 seconds is quite okay for a few changes, because one can relate time to output, but 20 seconds . . . And when processing a a few page document the waiting time of a second is often less than what one needs to move the mouse around to the viewer. Also, when a user starts $\text{T}_{\text{E}}\text{X}$ on the console and afterwards opens a browser from there that second is even less noticeable.

Now let's go back to improvements. A related addition was `\advanceby` that doesn't check for the `by` keyword. When there is no such keyword we can avoid pushing back the non-matching next token which is also noticeable. Here about 680 changes were needed. Changes like these only make a difference in performance for some very demanding mechanisms in $\text{ConT}_{\text{E}}\text{Xt}$. Again one cannot overload an existing primitive

because generic packages can fail (as the test suite proved). There were also a few places where a dirty trick had to be changed because we cannot alias these constants.

We can give similar stories about other improvements but this one sort of stands out because it is so noticeable. Also, other changes involve more drastic low level adaptations of ConT_EXt so these happen over a longer period of time. Of course all has to happen in ways that don't impact users. An example of a performance primitive is `\advancebyp1usone` which is actually implemented but still disabled because the gain is in hundreds of seconds range and I need to (again) adapt the source in order to benefit.

The mentioned register variants are implemented for count (integer), dimen (dimension), skip (gluespec) and muskip (mugluespec). Token registers are more complex as they have reference counters as well as more manipulator primitives. The same is true for boxes (although it is tempting to come up with some faster access mechanism) and attributes, that also have more diverse accessors. Also, token lists and boxes involve way more than a simple assignment or access so any gain will drown in other actions. That said, it really makes sense now to drop the maximum of 64K registers to some more reasonable 8K (or even less for mu skips). That will save a couple of megabytes which sounds like little but still puts less burden on the system.

14 LMTX on a phone

When my FairPhone 2 started to get issues (running hot and then rebooting) and some spare parts became hard to get, I moved on to a FairPhone 4. We're talking early 2022. The specifications of that little computer, which comes with a 5 year warranty and long term support are quite okay: a 1080x2340 pixel display, a Qualcomm SM7225 Snapdragon 750G (Octa-core (2x2.2 GHz Kryo 570 & 6x1.8 GHz Kryo 570), an Adreno 619 GPU, 8GB memory. an 256GB solid state disk, the usual phone gadgets like audio, camera, wireless, bluetooth and gps, and an USB Type-C 3.0 connector with support for OTG and DisplayPort.

Why do these specification matter? One reason is that in the compile farm we generate binaries for ARM processors and this phone has a decent one. The fast cores are in the same league as an over-clocked RaspberryPi 4 that we use in the compile farm for generating 32 bit binaries; the 64 bit binaries are generated in a virtual machine on a Mac Mini. So, in 2023, when looking at that phone, I wondered if we could run LMTX on it. I installed the UserLand linux stub from the Android Playstore and got myself an Ubuntu headless installation. After downloading the LMTX installer indeed I could install the distribution on the little machine.

A next step was trying to connect the phone to the display on my desk and after getting the right USB-C cable from the local computer shop I managed to get a bit larger terminal although Android 12 seems not able to use the whole 4K screen. Putting it in developers mode made it possible to enable the Android desktop interface in an external monitor. A bluetooth keyboard and mouse completed the setup. Later I tried a linux desktop but that was quite a disappointment so more research is needed there.

A predictable next step was to see if I could compile the LuaMetaTeX source that is part of the installation. Installing gcc and cmake was easy and indeed compilation went pretty well after that.

A quick performance test showed that making a format, which includes generating the file database, initially takes 10 seconds but less than 4 seconds once files are cached. Processing 1000 paragraphs from the `tufte` sample file is done with a reasonable 55 pages per second. I didn't test more complex documents but that might happen later, when the dock that I ordered has arrived, and when I have a decent display setup.

Given the fact that I only use a handful of applications on the laptop one can wonder when the moment is there that a properly dockable phone can do the job. Of course a disadvantage is that batteries are too small so one needs to provide power, but one needs a monitor, keyboard and mouse anyway. Wear and tear of the ssd can also be an issue but when storage is plenty that should work out all right. Of course it also assumes a stable operating system with one's favourite editing platform and viewer available.

15 Running green

There are a few contradicting developments going on: energy prices sky-rocket and Intel and AMD are competing for the fastest cpu's where saving energy seems mostly related to making sure that the many cores running at the same time don't burn the machine. However, $\text{T}_{\text{E}}\text{X}$ is a single core consumer so throwing lots of cores into the game is not helping much. You're better served with one very fast core than many slower ones that accumulate to much horsepower. The later makes sense when you process video or play games, but that's not what $\text{T}_{\text{E}}\text{X}$ is about, although it is fun to play with. Of course often multiple cores come in handy, for instance in the build farm that is used to compile LuaMeta $\text{T}_{\text{E}}\text{X}$ and intermediate $\text{T}_{\text{E}}\text{X}$ Live releases: when that gets compiled and we also trigger a LuaMeta $\text{T}_{\text{E}}\text{X}$ build, two times 10 linux virtual machines are compiling and one windows machine that runs four compile jobs at the same time.

The server that runs the farm is Dell 710 server with dual 5630 Xeon processors, 6 SAS drives each 2GB in (hardware) raid 10, 72 GB memory, redundant power supplies and 6 network ports. It sits idle for most of the time and consumes between 250 and 400W. It is part of a redundant setup: dual switches, dual routers, multiple UPS's, air conditioning, two backup QNAP NAS's, a few low power machines for distributed continuous incremental backups, etc. The server itself is a refurbished one, so not the most expensive, but with the Dutch energy prices of 2022 bound to gas prices, we quickly realized that there was no way we could keep it up and running. Because we have three such servers (one is turned off and used as fallback) we started wondering if we could go for a different solution.

As we recently upgraded the 2013 laptops to refurbished 2018 ones (the latest models that could use the docking stations that we have), we decided to buy a few more and test these as replacements for the servers. Of course one has to pimp these machines a bit: a professional 2TB nvme SSD plus a proper 2.5in SSD as backup one, 64 GB of memory, a few extra USB3 network cards. The cpu's are fast mobile Xeons. We use proxmox as virtual host and that runs fine in such a configuration.

Surprisingly, after moving the farm to that setup, which basically boils down to moving virtual machines, we found that running those parallel compilations performance wise was quite okay. And the nice thing was that these machines idle much lower, some 20–30W. The saving is therefore quite noticeable and we decided to check some more; after all it would be nice if we could bring down the average power consumption of 1750W down to at least half so that it would match the output of a few solar panels. Of course it means that one has to ditch perfectly well working machines which itself is not that environmental friendly but there is not much to choose here.

The second machine to be replaced was the one that runs quite some virtual machines too: the main file server, the mail server, an ftp server, the website, an rsync host, the squeezebox server that also serves as update test, and various project related rendering services. All run in their own (OpenSuse) virtual machine. After installing a similar laptop those were also moved.

As a side effect, the two backup NAS's were replaced by a single laptop (my 2013 Dell precision workhorse) running one backup file server, and for an extra incremental backup (rsnapshot running hourly, daily, weekly and monthly backups is our friend) a 2013 macbook was turned into a linux machine (15W idle with an internal reused SSD²¹ and an external 4GB disk), two managed switches became one (after all we had less network cables due to lost redundancy), only one backup power supply (that will be replaced by a nicer alternative when it breaks down; after all, by using laptops we get power backup for free). The total consumption went

²¹ For a change that apple machine was easy to update, and we could even get a new clone battery replacement.

down with at least 1000W. Of course there is an investment involved and we need to reconfigure the server rack, but the expectation is that by investing now we get less troubles later (less gambling on energy).²²

But, there is still the pending question of what the impact is on the services that we run. The most demanding ones are the Math4all and Math4mbo: these produce large files, need many resources (xml and images), and we didn't want to burn ourselves too much. Now, here is an interesting observation: this service runs twice as fast on the new infrastructure. But it is hard to explain why. The file server is on a different machine (so no fast internal network), the cpu is a bit faster but not that much, the virtual machine is on ssd, but files are saved on the file server, which is a two disk usb3 enclosure connected directly to a virtual machine that does software raid. The most important difference is that main memory is much faster and T_EX is a memory intense process. From when we started with LuaT_EX we do know that memory bandwidth and cpu caches makes a difference. Maybe the faster floating point handling for the more modern Xeon also helps here.

And that brings me to the following: how do we actually benchmark T_EX? When you go on the internet and compare cpu's most tests are not that comparable to a T_EX run on a single core. One can think of a set of test files, but the problem there is that when the engine evolves and details in the macro package coding changes, one loses the comparison with older tests. This is why, when we do such tests, we always run the same test on the different platforms. Although this often shows that the gain on newer hardware is seldom what one expects from the more general benchmarks, one can still be surprised. When we moved to five year newer laptops the gain was some 30% for me and 50% for my colleague. The difference between his laptop and the slightly more beefed up virtual machine can be neglected.

We monitor the power consumption with a youless device connected to the power meter. When I process the LuaMetaT_EX manual I see the phase that the machine sits on go up 20W for a run that takes some 9 seconds. Let's say that we use 180Ws or 0.0006kWh (20.000 runs per kWh). So, compared to the idle power usage of a server, a single T_EX run can be neglected, simply because it is so fast. So, what is actually the most efficient hardware for a T_EX service? I get the feeling that a decent Intel Atom C3955 16-Core driven machine is quite okay for that, but I don't have that at hand and last time I checked one could not order anything anyway. And with prices of hardware going up it's also not something you try for fun. As comparison to what we have now, testing T_EX on an Intel NUC11ATKC2 could also be interesting (it has an N4505 cpu). There was a time when I considered a bunch of raspberry pi's but they no longer are that cheap, given that you can get them, and adding a case and proper disc enclosure also adds up. When wrapped in a nice package the pi will probably a couple of times slower but it then probably also uses less power. These fitlets are also interesting but again, one can't get them.

It is kind of fun to play with optimizations that don't really impact the clarity of the code. One can argue that spending a day on something that saves 0.005 seconds on a specific run is a waste of time, but of course one has to multiply that number by a number of runs. Personally I will never gain from it but nevertheless it can save some energy: imagine a batch of 15000 documents every day. We then save $\frac{15000 \times 0.005}{3600}$ seconds or about 8 hours runtime. This can still be neglected but what if this is not the only optimization?

An example of such an optimization is this:

```
\advance\somecounter \plusone
\advance\somecounter by \plusone
```

²² We hope to save some 9000 kWh which means that save at least some 2500€ per year and more when the government will reinstate its energy tax policy and or prices go further up, which seems to be the case. Even before the crisis in the Netherlands 5ct/Kwh became fives times that amount effectively when connection, transportation, energy tax and value added tax gets added.

The second one runs faster because there is no push back involved as side effect of the lack of a keyword, so how about adding this to the engine?

```
\advanceby \somecounter \plusone  
\advancebyone\somecounter
```

Given the way LuaMetaTeX is coded, it only needs a few lines! In this case it extends the repertoire of primitives so it is visible but we have many other (similarly small) optimizations that contribute. Again, the average user will not notice a drop in runtime from 1.5 seconds to 1.45 but when 8 hours become 80 hours or 800 hours it does become interesting. In energy sensitive 2022 these 800 hours not only save some €400 but also contribute to a lower carbon footprint! And now imagine how much could be saved on these extensive runs when we make sure that the style used is optimal? Of course, when we need two runs per document it starts adding up more.

Some experiments with a demanding file showed one percent gain (on a 2.7 seconds run) using the alternative integers, dimensions and advance primitives. However, using ConTeXt's compact font mode brought down runtime to 2.0 seconds! So, in the end it's all very relative. It is worth noticing that the .7 seconds saved on fonts is sort of constant, which means that accumulated gains elsewhere makes that .7 seconds more significant as we progress.

16 Supporting math in the JMN collection

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Introduction

In 2022 we overhauled math font support in ConT_EXt, using new functionality in the LuaMetaT_EX engine. By that time it had become clear that the OpenType math font landscape had more or less settled. The Latin Modern fonts as well as the T_EXgyre fonts don't evolve, so we consider them being frozen. It is also unlikely that the reference Cambria font will change or become more complete. More recent math font are modelled as a mixture of Cambria and Latin Modern.

When we started with LuaT_EX in ConT_EXt we immediately started using Unicode math but the lack of proper Unicode fonts, with the exception of Cambria, resulted in creating virtual Unicode math fonts on the fly using the virtual font features of the LuaT_EX. But when the OpenType math fonts came available that kind of trickery was no longer needed (or at least less preferred).

That is why we considered dropping the virtual math font mechanism from LMTX. We had already dropped **tx** (we can use Termes) and **px** (better use Pagella) fonts as well as Type1 based Latin Modern. Dropping the commercial Math Times was a logical next step, also because it has never been tested. The mixtures of Pagella and Euler were already replaced by using the upgraded tweak mechanism.

That left us with Antykwa, Iwona and Kurier, the fonts that the late Janusz Nowacki vectorized and that came with plenty OpenType text fonts but also with Type1 math companions. And, as we like these fonts, it meant that we had to come up with a solution. One option was to create proper OpenType math fonts, but another was to strip down the virtual math font mechanism to just support these fonts. There is some charm in keeping the Type1 fonts, also because it is a test case for (by now sort of obsolete) tfm metric, pfb outline, encoding and map files, for which we have code embedded so having a proper test case makes sense.

In the end we opted for the second solution so this is what the next sections are about: supporting OpenType math using Type1 fonts. We admit that it took way more time than a conversion to OpenType math fonts would have, but that is partly due to the fact that these fonts, and especially Antykwa, have some characteristic features that we wanted to use. So, in a sense it was also an esthetics challenge. It also helped that the font was used in realistic and moderately complex math rendering. We also note that rendering in LMTX is different (and hopefully better) than in MkIV because we try to benefit from the upgraded math engine in LuaMetaT_EX.

This exploration is dedicated to Janusz who was one of the characteristic presenters of fonts at BachoT_EX meetings, who contributed these fonts, and who in some sense was thereby kick starting the Polish T_EX related font projects.

Virtual math

We keep this expose simple and only tell what we did, you can look at the **lfg** files and source code to see what magick is done. For the standard Antykwa we load **LatinModern-Math** first, so we cover all symbols that matter. Next we stepwise load **rm-anttr.tfm**, **mi-anttri.tfm**, **mi-anttbi.tfm**, **rm-anttb.tfm**, **sy-anttrz.tfm**, **ex-anttr.tfm**. For each we specify an encoding vector. Some are loaded multiple times with different vectors. Because we don't like the slanted curly braces we even load **AntykwaTorunska-Regular** in order to get the upright ones. This method is not that different from what we do in MkIV.

The specification of a loaded font also can contain a list of (named) characters that should be ignored, That was one of the new features in the virtual constructor. We take the math parameters from the fonts where these are specified, here in the symbols and extension fonts.

The fonts contain extra snippets of extensibles that one can use to construct some of these vertical and horizontal stretched symbols on the fly in addition to what the font metrics already define. Unfortunately some snippets are missing, like the six pieces that could make up horizontal and vertical bars, for which we now need to cheat. We considered making a companion font but for now we are in ‘as good as we can’ emulation mode.

The fonts

We start out with a skeleton font and in the past we used the OpenType text font for that. On top of that we overlay a bunch of Type1 fonts, and as was common in those days, the ams math symbol fonts `msa` and `msb` were overlayed last in order to fill in remaining gaps. However, now that we have a Latin Modern OpenType math font it made more sense to use that as starting point because it already has all these symbols.

If we forget about the additional weights and condensed variants, the JMN math collection has actually not that many fonts. One reason for that is that the upright roman font, the ones that have an `r` near the end of the file name, in traditional \TeX speak `rm`, have way more than 255 characters: it not only has all kind of composed characters, it also has all the extensible shapes. All is (as usual with Type1 fonts) driven by encoding and mapping files. Fortunately the glyphs names (that we use for filtering) are the same for the three fonts but there are some more in Antykwa.

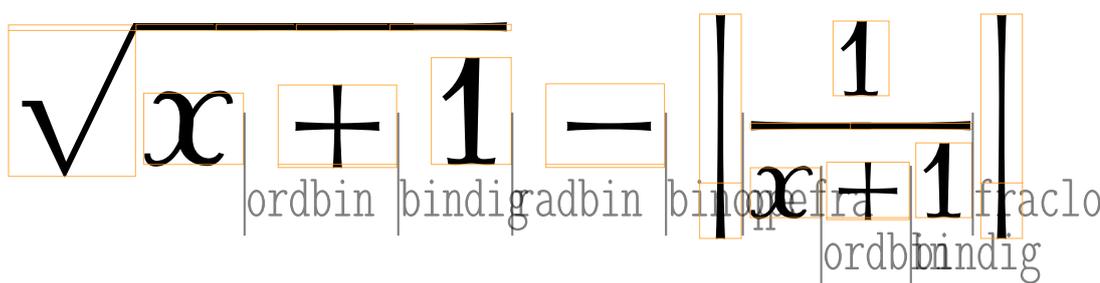
Challenges

The real challenge was Antykwa. This is because it has a distinctive curvature at the end of sticky parts (like rules and such). The \TeX machinery as well as OpenType math assume rules being used in for instance radicals, fractions, overbars, underbars and vertical bar fences.



The last three come in not only sizes (aka variants) but also can stretch (aka extensible). And, them being just rules it is assumed that the \TeX engine deals with that, and as it cannot really do that without characters, the traditional approach is to use commands that use \TeX rules. However, in Con \TeX t (MkIV and LMTX) we can provide the proper variants and extensible using virtual shapes, and in LMTX we can even scale as last resort.

The first two are special. We already could support fractions using dedicated characters because we played with the fraction builder using for instance arrows as separator and these are not rules but characters. It was not that much work to also make that possible for the rule in a radical. The main adaptation was that we need to center the numerator and denominator of a fraction and the body of a radical when the character used is wider than requested.



Because we have two font models in ConT_EXt, normal and compact, we had to be careful in defining the virtual shapes and extensibles so that they work in both models. This has to do with scaling and sharing.

Implementation

The original virtual math font mechanism worked closely with the math fall back features, but these have been replaced by tweaks, which means that we lost some of that. Also, heuristics worked fine on the average but for Antykwa we wanted more. Therefore some of the built in logic has been moved to the goodie file that controls the composition. After all, we don't want to hard code specific solutions in the core.

Another addition was the use of so called setups bound to a font class so that we can set up some math machinery features for e.g. Antykwa in the goodie file. We need to bind to a class because we mix a dozen math fonts in one test file and therefore we need to separate these setups.

As in regular OpenType math support we ignore the italic correction and translate it in combinations of proper width and specific kerning. That way we avoid all kind of issues that we otherwise need to compensate for.

Because we need to hook extensible characters into the machine for Antykwa its font typescript file also defines a font class specific setup (of a few lines) to be applied. This might evolve into a more granular mechanism but for now it works fine and adds little overhead.

Examples

We end by showing a few “real” examples.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad 0 \leq k \leq n.$$

The parentheses are unchanged, and we believe that using rotation symmetry instead of mirror symmetry is a brave but interesting choice, and the new fraction bar fits well with it. The fraction bar also fits well with the equal sign.

$$1 + x + x^2 + \dots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

The new vertical bars go well with the brackets, the integral and the solidus.

$$\|f\|_p = \left[\int_0^1 |f(x)|^p dx \right]^{1/p}$$

The fancy fraction bar and the radical bar have made the arithmetic-geometric mean inequality look more appealing than ever, hasn't it?

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n}$$

Primes are usually a bit of a challenge:

$$\begin{aligned} f(k+1) - f(k) &= f'(k) + f''(k) \frac{1}{2} + f'''(\xi_k) \frac{1}{6} \\ &= k^p + \frac{p}{2} k^{p-1} + \frac{p(p-1)}{6} \xi_k^{p-2} \end{aligned}$$

And, as expected, multi-line formulas also look fine.

17 Profiles

17.1 Introduction

Among the typesetting problems that relate to math are inline formulas that have a bit too much height or depth but not so much as to justify some additional interline space. For that reason in ConT_EXt MkII we have some snapping features that can be enabled that limit the dimensions. In MkIV a more extensive profile feature was written (we talked about it at meetings in 2015) that look at the bottom and top of lines in order to determine if lines can be moved closer, but in practice snapping and profiling are never really used. In the end it was more an academic exercise which is not uncommon when it comes to T_EX user demands and practice.

As part of exploring math micro typography these features surfaced again during some discussion about weird mechanisms and we actually wondered if we could revive this now that we also control other aspects of math typesetting in more detail. One condition is that the overhead is not that high. Users accept some overhead for protrusion and expansion that relate to horizontal optimization so a little extra overhead for vertical optimization should not be a problem.

Of course, as with protrusion and especially expansion, the question is if readers will notice it. Best would be to set up some experiments but, although one can argue that research is important, in practice it always boils down to a visual impression, feel good and, like it or not, exploration, trial and error. And so a simplified variant of profiling was implemented and applied to a math intensive math book used in academia. Instead of proper experiments some unaware bystanders were asked if they noticed a difference and to our surprise that was the case! And these were not even math students but kids who were more familiar with children books and phones. That convinced us that we were on the right track, that we need to explain a little about what we actually do, and that we should tell users what to look at when this gets applied.

With an introduction like this, mentioning ‘research’, ‘academia’, ‘students’ and ‘typography’ we’re sure that future generations will be convinced that what is discussed next has a strong fundament so here we go!

17.2 A first example

We start with a simple example. Because in practice profiling always kicks in when possible one really need to handcraft an example that can be used for demonstration: figure 17.1.

In order to see what happens it is important to understand how T_EX sees lines. Actually, the concept of lines in T_EX is rather limited: lines are just horizontal boxes where the baselines are separated by `\baselineskip` and when the distance is larger than that dimensions a `\lineskip` gets added.

```
these are a few .....
lines of text .....
where lines have depths .....
or no real .....
height at all .....
```

Between all these lines some skip needs to be added: the `\baselineskip` minus the height and depth. If we add struts the lines get the optimal height and depth so then no skips are inserted:

```
these are a few .....
lines of text .....
```



Figure 17.1

where lines have depths
or no real
height at all

When we increase the depth a little, for instance 1.2 times the normal strut depth, we see that some additional space, the `\lineskip` gets added:

these are a few
lines of text
where lines have depths
or no real
height at all

However, there is no real reason to do that here because the larger rules don't clash with text or other content. So this is what we get when we pass the `profile` option to `\setupalign`:

these are a few
lines of text
where lines have depths
or no real
height at all

Profiling works per paragraph, so when we add a `\par` in the middle we get this:

```

these are a few
lines of text
where lines have depths
or no real
height at all

```

But, we can actually setup the profiler to look back. Setting up the main (document) profiler happens with:

```

\setuplineprofile
  [factor=0.125,      % default
   paragraph=yes,    % default: no
   step=0.5\emwidth] % default

```

but as with most ConTeXt mechanisms you can define your own profiler. The step tells what granularity to use when comparing positions in a line. The factor sets the threshold for the interline skip. We saw these two differ in the first example we gave.

17.3 Profiled math

We will give several examples of math profiling. In the examples we will switch font to Latin Modern, since the effect is more visible for that font. Most of our examples will be “real” (slightly modified) ones, but we start with a rather artificial example. Below we have two occurrences of a fraction. Note that the profiling only kicks in for the second one. The reason is that on the line above the first one we only have letters (x) with no depth, while in the second one, we have added one letter (g) that has depth.

```

XXX XXXX XXXX XX XXX XX XX XXX XXXX XXX XX XXX XXX XX
XX XXX X XXXXX XXXXX XXX XXXX XXXX XX XXX XX XX XXX
XXXX XXX XX XXX XXX XX XX XXX X XXXXX XXXXX ... XXX XXX
XX XX XXX XXXX XXXXXXXX XXXX XXXX XXXXX XXX XXXX XXX
XXX XXX XXX XXX XXX XXXX XXXXXXX XXXXXXX XXXgXXX XXX
XXX XXX XX X XXXXXXX XXX - XXX XXXX XXXX XX XXX XX XX
XXX XXXX XXX XX XXX XXX XX XX XXX X XXXXX XXXXX.

```

no profile

```

XXX XXXX XXXX XX XXX XX XX XXX XXXX XXX XX XXX XXX XX
XX XXX X XXXXX XXXXX XXX XXXX XXXX XX XXX XX XX XXX
XXXX XXX XX XXX XXX XX XX XXX X XXXXX XXXXX ... XXX XXX
XX XX XXX XXXX XXXXXXXX XXXX XXXX XXXXX XXX XXXX XXX
XXX XXX XXX XXX XXX XXXX XXXXXXX XXXXXXX XXXgXXX XXX
XXX XXX XX X XXXXXXX XXX - XXX XXXX XXXX XX XXX XX XX
XXX XXXX XXX XX XXX XXX XX XX XXX X XXXXX XXXXX.

```

step=1pt, factor=0.125

We next show a simple paragraph where the mechanism gets applied in three out of five line breaks.

```

The results of Section 6.3 show that the same phenom-
enon is encountered when treating the norms of in-
verses: converges to very fast if ,
while the convergence may be slow if . As the
following proposition reveals, at least for the strict
inequality is the generic case.

```

no profile

```

The results of Section 6.3 show that the same phenom-
enon is encountered when treating the norms of in-
verses: converges to very fast if ,
while the convergence may be slow if . As the
following proposition reveals, at least for the strict
inequality is the generic case.

```

step=1pt, factor=0.125

We have shown the lines and used the helper to show where the profiling is applied. We show the same example but without these helpers. After all, this is how we usually see it.

The results of Section 6.3 show that the same phenom-
 enon is encountered when treating the norms of in-
 verses: converges to very fast if ,
 while the convergence may be slow if . As the
 following proposition reveals, at least for the strict
 inequality is the generic case.

no profile

The results of Section 6.3 show that the same phenom-
 enon is encountered when treating the norms of in-
 verses: converges to very fast if ,
 while the convergence may be slow if . As the
 following proposition reveals, at least for the strict
 inequality is the generic case.

step=1pt, factor=0.125

If the paragraph is slightly reformulated, the profiling might change. Below we show an example where the subscript (p) on the fourth line gets too close to the superscript (0) on the last line.

The results of Section 6.3 show that the same phenomenon is encountered when treating the norms of inverses: $\|x\|_p$ converges to $\|x\|_0$ very fast if $\|x\|_p$ is small, while the convergence may be slow if $\|x\|_p$ is large. At least for $\|x\|_p$ the strict inequality $\|x\|_p < \|x\|_0$ is the generic case.

no profile

The results of Section 6.3 show that the same phenomenon is encountered when treating the norms of inverses: $\|x\|_p$ converges to $\|x\|_0$ very fast if $\|x\|_p$ is small, while the convergence may be slow if $\|x\|_p$ is large. At least for $\|x\|_p$ the strict inequality $\|x\|_p < \|x\|_0$ is the generic case.

step=1pt, factor=0.125

We can configure the amount of space that shall be added with the `factor` key.

The results of Section 6.3 show that the same phenomenon is encountered when treating the norms of inverses: $\|x\|_p$ converges to $\|x\|_0$ very fast if $\|x\|_p$ is small, while the convergence may be slow if $\|x\|_p$ is large. At least for $\|x\|_p$ the strict inequality $\|x\|_p < \|x\|_0$ is the generic case.

step=1pt, factor=0.125

The results of Section 6.3 show that the same phenomenon is encountered when treating the norms of inverses: $\|x\|_p$ converges to $\|x\|_0$ very fast if $\|x\|_p$ is small, while the convergence may be slow if $\|x\|_p$ is large. At least for $\|x\|_p$ the strict inequality $\|x\|_p < \|x\|_0$ is the generic case.

step=1pt, factor=1

This shows that the sequence $\|x\|_p$ is a Cauchy sequence in \mathbb{R} and therefore it converges in norm to some $\|x\|_0$, by Theorem 2.8.1. On the other hand, by Theorem 2.8.2, there exists an increasing sequence of positive integers n_k such that $\|x\|_{n_k} < \|x\|_0$ almost everywhere.

no profile

This shows that the sequence $\|x\|_p$ is a Cauchy sequence in \mathbb{R} and therefore it converges in norm to some $\|x\|_0$, by Theorem 2.8.1. On the other hand, by Theorem 2.8.2, there exists an increasing sequence of positive integers n_k such that $\|x\|_{n_k} < \|x\|_0$ almost everywhere.

step=1pt, factor=0.125

17.4 Line spacing

When we enable the line profiler on a 300 page math course with plenty inline formulas, the number of ‘corrections’ varies a lot with the fonts. Some simple tests show that Latin Modern, Bonum and EBGaramond get quite some applied, while Lucida, DejaVu, Antykwa, Erewhon and Libertinus only see a few corrections. Pagella, Termes and StixTwo end up in the middle.

The trigger is not always text or math. The course material has quite some structure, like numbered descriptions. In ConT_EXt we use plenty of struts to make sure that spacing is consistent and the keyword that starts a description therefore gets them. Normally that is not an issue but when the height of the next line exceeds the strut height we get a clash and line skip will be added. One can argue that the strut spoils the typesetting but in general it does more good than harm, at least in ConT_EXt. It looks like the profiler is quite capable of getting rid of the cases where it interferes (or more precisely: where it doesn't run into the next line).

The reason why we get a line skip added is simple: when the depth of the first line equals strut depth and the height of the second one equals strut height we're okay. When one of them is less we're also okay because T_EX will adapt the baseline skip so that it compensated the difference. However, when the first line has strut depth (due to the present strut) and the second line more than strut height (resulting for instance from a formula) the lines are considered overflowing in each other and therefore interline skip gets added.

When we end up in this situation the profiler can bring down the line skip when it concludes that the strut is not running into the next line. However when the formula sits directly below the strut we cannot really

determine what is right so then we just keep the skip. This situation occurs seldom. In many cases struts are optional so one can always disable them (locally).

As the mentioned test document uses Lucida as body font, in the three cases where we actually get a clash, one definitely relates to the strut: the overflow in the second line occurs close to the right margin and the strut in the first line sits at the left margin so we can get rid of the line skip, which leave us with only two cases. However, there is another observation, one that involves the baseline distance or line height.

In ConT_EXt the ratio between the strut height and depth is 72:28 which works quite well for most fonts. If we look at Lucida shapes we see that the depth is normally small so we can actually decide to change that ratio. It is however not clear how that will influence decisions. Assuming more height will help with for instance formulas that have superscripts, but an inline integral with subscript might suffer. For other fonts, like EBGaramond, that have some extremely deep shapes changing rations won't help anyway. We win here and loose there.

We will look into a few fonts to get a better impression how all this relates. We will use 10pt sizes. When we compare Lucida, Latin Modern, Bonum and Pagella we notice that we start out with design sizes that are quite different.

lucida

strut ht : 10.67896pt
strut dp : 4.15291pt
ex : 5.29709pt

modern

strut ht : 8.68419pt
strut dp : 3.37718pt
ex : 4.30763pt

bonum

strut ht : 9.77223pt
strut dp : 3.80031pt
ex : 4.84734pt

pagella

strut ht : 9.44986pt
strut dp : 3.67493pt
ex : 4.68742pt

This is why we always use ratios (the 0.72 and 0.28) as well as abstract dimensions like `ex` and `em` so that we adapt to what the font provides. Because the default (total) line height is set to `2.8ex` we get larger values in for instance Lucida.

In the next tables we use three samples, with `text-001` being:

abcdefghijklmnopqrstuvwxy
ABCDEFGHIJKLMNOPQRSTUVWXYZ
1234567890
() [] / \ { } |
. , ! ? @ # \$ % ^ & * _ - + =

The two math samples are:

$\int \sum \sqrt{\quad}$

and

x_2^2

For text the ratios are not that far off the defaults, but for math they start to differ and the distance becomes larger. For Lucida we get:

text-001		math-001		math-002	
0.79	7.8257pt	0.81	9.78223pt	0.92	7.40593pt
0.21	2.04887pt	0.19	2.24377pt	0.08	0.62965pt

Modern gives:

text-001		math-001		math-002	
0.75	7.49588pt	0.75	9.16898pt	0.84	7.43591pt
0.25	2.49863pt	0.25	3.05333pt	0.16	1.37924pt

And bonum moves in the other direction:

text-001		math-001		math-002	
0.77	7.90565pt	0.77	10.16666pt	0.82	7.21603pt
0.23	2.40868pt	0.23	3.11829pt	0.18	1.54915pt

Pagella also differs:

text-001		math-001		math-002	
0.73	7.49588pt	0.78	10.69714pt	0.85	6.88622pt
0.27	2.82845pt	0.22	2.95837pt	0.15	1.24931pt

Changing the ratios for the sake of math makes not that much sense because profiling depends a lot on what math ends up inline. When we looked around a bit for realistic examples we got the impression that seeing some clash (read: getting uneven line spacing) might be a reason why small formulas eventually end up as display. Without mentioning names, we noticed that a reprint of a book actually got reformatted and when we looked for the slashing formulas in the original they had become display instead. With proper profiling there is no need for that. On top of that one can argue that some inline rendering can be done better anyway, like using skewed fraction instead of ruled ones. Can we predict that math with many superscripts goes well with a font with relatively high shapes? As soon as some parenthesis are used we get depth anyway and how likely is it that math without these is used inline? It also depends on the amount of math: two lines with superscripted math will drive $\text{T}_\text{E}\text{X}$ to use line skip so we need to profile anyway. For that reason we will not adapt the ratios, just like we keep the default line spacing. There are of course fonts with extreme heights (like the Computer Modern Dunhill variant) but no one will use those artistic variants in a math document. If you want to go fancy and distinctive, Antykwa is a good choice and that one actually scores pretty good with the defaults!

antykwa

strut ht : 9.47pt
strut dp : 3.68277pt
ex : 4.69742pt

text-001		math-001		math-002	
0.75	7.69577pt	0.76	9.59224pt	0.87	7.0961pt
0.25	2.49863pt	0.24	3.05562pt	0.13	1.04942pt

17.5 Conclusion

So, should we enable profiling on mixed text-math documents or not? One possible reason for not doing it is that it adds overhead, but in practice it's not that much compared to processing the rest. It is no problem to find complaints on the internet about Lua \TeX performing worse than its ancestors so if you're in that category: don't use profiling because it sets you back a few percent runtime. However, when you're a demanding Con \TeX t user who mixes in a lot of math, you might give it a try. It will intercept a couple of cases where struts (assuming structure is used) trigger a line skip, and it might also catch a couple of cases where \TeX found lines getting too close. Tweaking the **factor** and **step** can actually be fun. Because it does influence page breaks it is not something to be applied last minute. And, talking performance, this kind of vertical optimization comes cheaper than horizontal optimization using expansion (hz) and protrusion.

18 Pushing the envelope

Here I describe the results of some exploration and experiments by Mikael Sundqvist and me. We got side-tracked from intersections, arcs and drawing functions when we noticed some artifacts with envelopes. But what are envelopes actually? Let us start with a simple path:

```
\startMPinclusions
  path TestPath ; TestPath := fullcircle xyscaled (10cm,1cm)
\stopMPinclusions
```

When we draw this with a circular pen we get this:

```
\startMPcode
  draw TestPath withpen pencircle scaled 2mm withcolor darkred ;
\stopMPcode
```



Filling gives:

```
\startMPcode
  fill TestPath withpen pencircle scaled 2mm withcolor darkred ;
\stopMPcode
```



When a `pencircle` is used MetaPost delegates the work to the backend because PostScript has a circular pen, otherwise it has to calculate the to-be-filled shape itself. The backend has to do some path juggling in the case of pdf because there a pen transform is different from PostScript.

```
\startMPcode
  draw TestPath withpen pensquare scaled 2mm withcolor darkblue ;
\stopMPcode
```



Here we draw the shape with a square pen while filling gives:

```
\startMPcode
  fill TestPath withpen pensquare scaled 2mm withcolor darkblue ;
\stopMPcode
```



In most cases this works out well but there are some hidden issues. These get exposed when we use a transparency:

```

\startMPcode
  fill TestPath withpen pensquare scaled 2mm withcolor darkgreen
    withtransparency (1,.5) ;
\stopMPcode

```

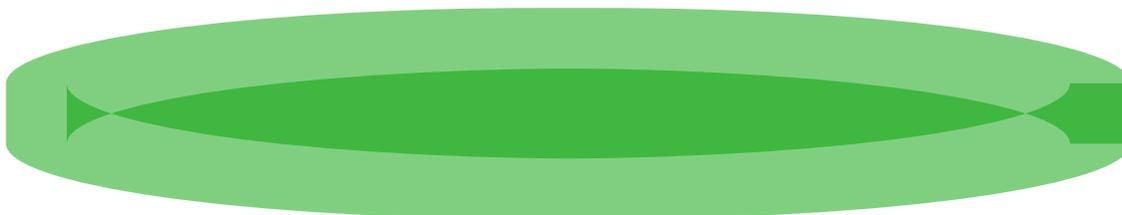


It are these artifacts that we will explore a little. For that we will render quite some graphics. We could show numerous more examples but when you are a ConT_EXt user you will be able to make plenty yourself by looking at these examples.

```

\startMPcode
  fill fullcircle xyscaled (.8TextWidth,2cm)
    withpen pensquare scaled 8mm
    withcolor darkgreen
    withtransparency (1,.5) ;
\stopMPcode

```



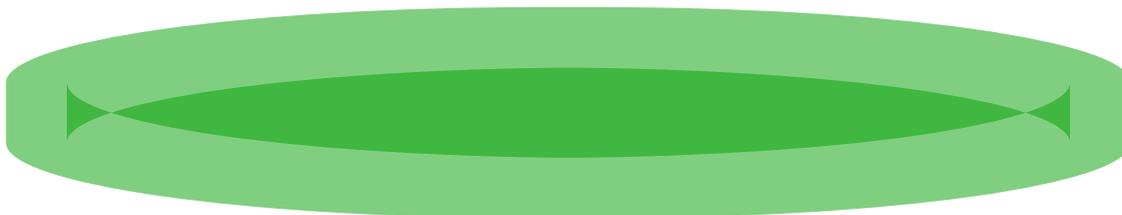
When we were playing with the `envelope` primitive we noticed these artifacts and we spent quite some time looking at the code to see where it comes from and if we could prevent this. It was then that we realized that the fill actually also uses these envelopes but that it gets delayed till the shapes are flushed to the backend. That meant that we could use fills with transparencies as simple test cases.

The first thing to get rid of is the weird blob at the right end of the fill in this example. Not really understanding all what went on, we explored all kind of shapes and temporarily disabled some of the code in the Meta-Post library to see where it crept in. We decided that touching the code to get rid of for instance rounding issues or potential direction related side effects made no sense. In the end the solution was simple:

```

\startMPcode
  pen p ; p := makepen(unitsquare rotated eps) ;
  fill fullcircle xyscaled (.8TextWidth,2cm)
    withpen p scaled 8mm
    withcolor darkgreen
    withtransparency (1,.5) ;
\stopMPcode

```

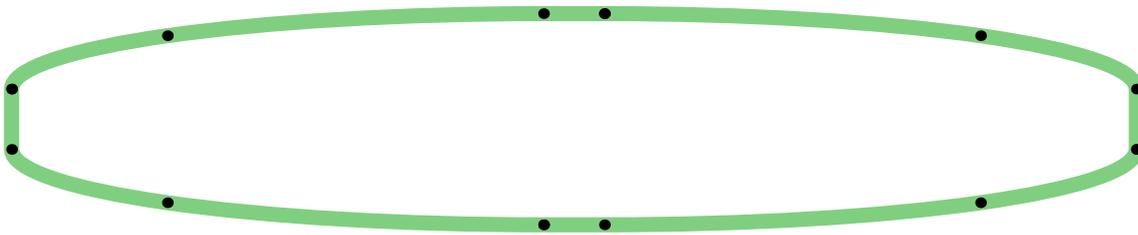


We show what the `envelope` primitive gives us:

```

\startMPcode
  pen p ; p := makepen(unitsquare rotated eps) ;
  path e ; e :=
    envelope (p scaled 8mm)
  of
    (fullcircle xyscaled (.8TextWidth,2cm))
  ;
  draw e
    withpen pencircle scaled 2mm
    withcolor darkgreen
    withtransparency (1,.5) ;
  drawpoints e ;
\stopMPcode

```

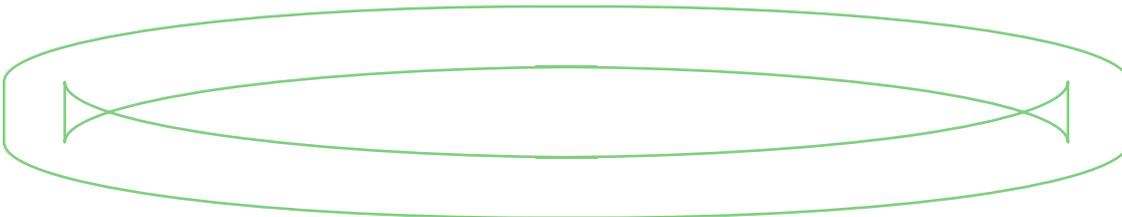


This looks okay compared to previous the examples but we have only a simple path here, while the fill actually has two:

```

\startMPcode
  pen p ; p := makepen(unitsquare rotated eps) ;
  enfill fullcircle xyscaled (.8TextWidth,2cm)
    withpen p scaled 8mm
    withcolor darkgreen
    withtransparency (1,.5) ;
\stopMPcode

```



So how do we get that inner shape? Once you know what a fill actually outputs to the backend it is easy! There are two envelopes: the normal one and one made from the reverse path (or in internal MetaPost speak: htap). In the previous example the `enfill` treats the path as a fill but will draw the envelopes instead. As with `eofill`, `eoclip` and path accumulators this is a MetaFun backend related feature but we introduced `enfill` as a new one.

```

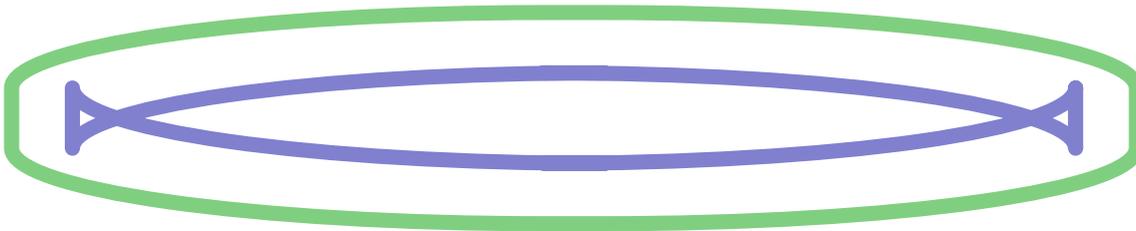
\startMPcode
  pen p ; p := makepen(unitsquare rotated eps) ;
  draw
    envelope (p scaled 8mm) of
      (fullcircle xyscaled (.8TextWidth,2cm))
    withpen pencircle scaled 2mm

```

```

withcolor darkgreen
withtransparency (1,.5) ;
draw
  envelope (p scaled 8mm) of
  reverse (fullcircle xyscaled (.8TextWidth,2cm))
  withpen pencircle scaled 2mm
  withcolor darkblue
  withtransparency (1,.5) ;
\stopMPcode

```



We're now ready for the real deal but keep in mind that what we show here is the result of stepwise growing insight combined with adding some features to the engine that not only makes it possible to illustrate this but also might prove to be useful. The used primitives will be explained later, for now we just stick to the results.

Figure 18.1 shows a circle filled (or enveloped) with pens made from `fullcircle`, `fulldiamond`, `fulltriangle` and `fullsquare`. The paths that we use for the pens are also shown. The outcome can be puzzling but after going over the code (in the engine) and trying to reason the logic it becomes clear that the unexpected is mostly due to the fact that there is no other way to draw the path (read: meet the criteria).

When looking closely at the results (adding labels to the points and zooming in) one will notice more side effects. Because we rotate over `eps` to get rid of the weird end situation we can end up with more points than we like and these are so close to each other that one doesn't notice them. For this we can apply the scrutinizer:

```
e := e scrutinized 0.01 ;
```

When that is done we can wonder if a simplified (inner) path is possible. I tried a few solutions using the Lua interface while Mikael (as mathematician) followed the more scientific approach but the results largely depend on the pens and shapes.

Actually when doing all that we used a more complex pen in several variants. This is shown in figure 18.2. Notice the dashed lines here. When a pen is defined there is some checking going on. One is that circular pens get no treatment at all and just pass through the system. Basically any single point cycle is considered as elliptical anyway. Then the path turned into a so called 'convex' path. It also showed us the real pen being used. When out of curiosity I commented that bit of code I noticed that we could achieve interesting results. The result is that we now have a `convexed` primitive. After all the code was there so it took only a few lines to add this primitive. In figure 18.3 you can see the result of a unconvexed pen.

We can also calculate envelopes of non-cyclic paths which is demonstrated in figure 18.4 and figure 18.5. There is however some trickery involved. Just to make this easier the MetaFun macro package has a type starring macro that makes such a star:

```
path p ; p := starring(-1/2) rotated eps ;
```

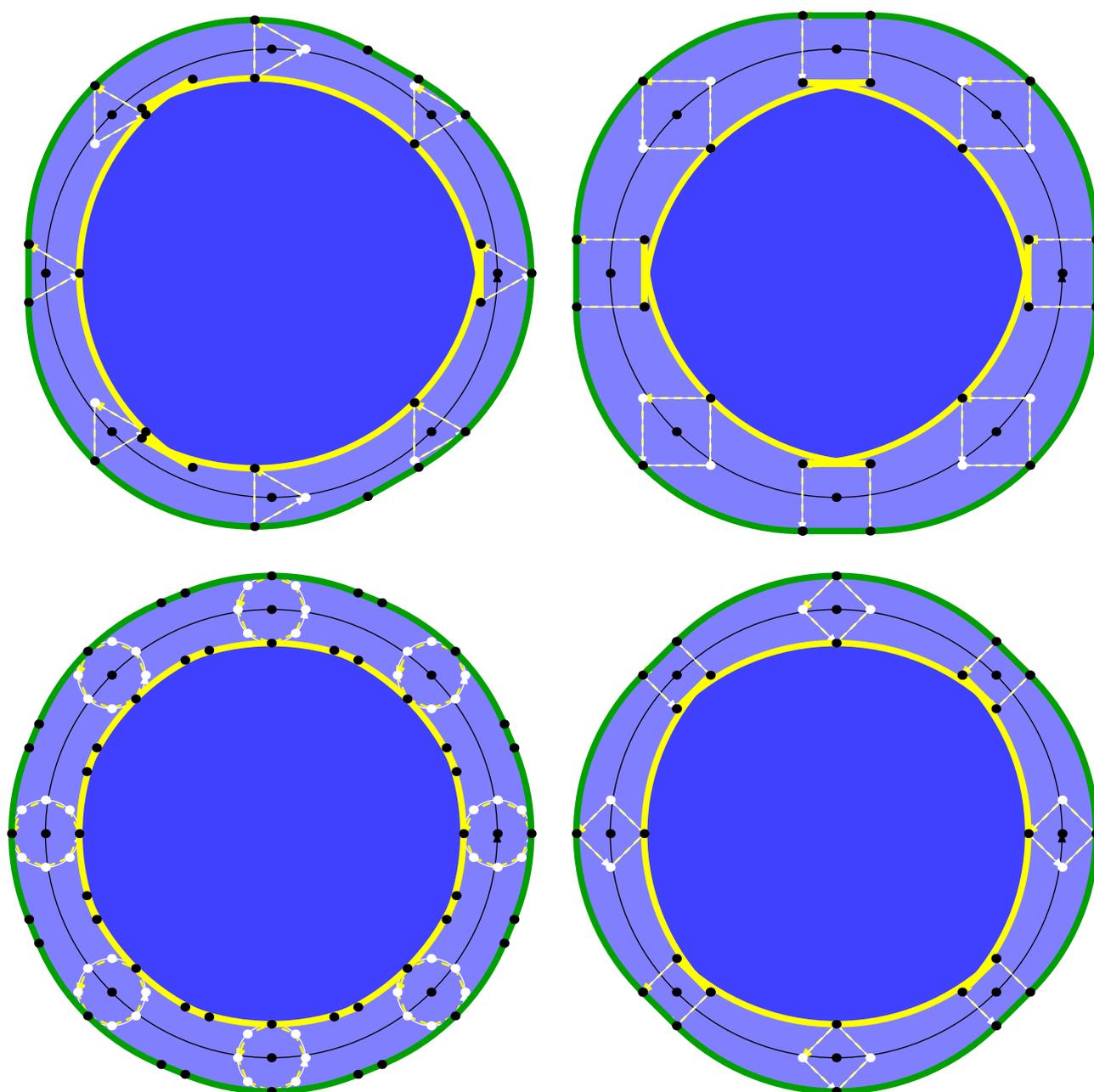


Figure 18.1 Using four different relatively large pens on a circle.

This star can become a pen:

```
pen somepen ; somepen := makepen (pp) ;
```

And as mentioned pens get convexed by default. Even worse, whenever we transform a pen it gets convexed again. When we fill a shape the pen gets attached to that shape and the backend will do the enveloping. The easiest way to consistently avoid convexing was to introduce a new pen type.

```
nep somepen ; somepen := makenep (pp) ;
```

The somewhat weird short `nep` perfectly fits the bill as in Dutch it means fake. A pen defined this way stays unconvexed. Actually there is another property where pens differ from regular paths: they are double linked. In original MetaPost that back (prev) link uses a field in a knot record that is not used by pen paths. The path that gets pencilled also abuses one of knot fields for keeping track of the offset that a point has relative to the current point in the pen. It was good moment to also make regular paths double linked lists.

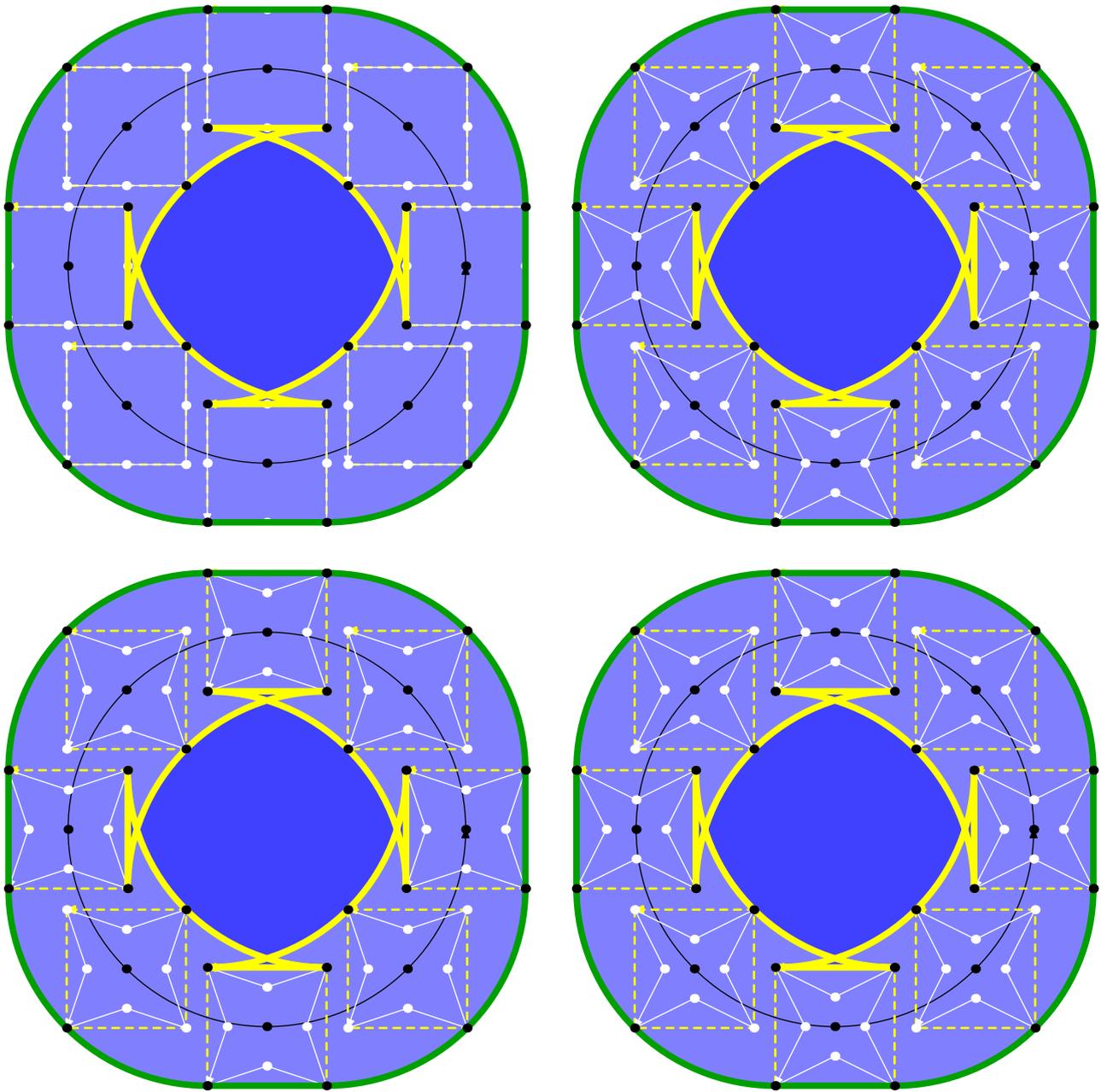


Figure 18.2

That comes at the cost of an extra pointer in the knot record but we could also save some space by using smaller slots for other fields. Memory is not our biggest worry anyway.²³ Double linking meant that there was no need for doing that when making pens.²⁴

We can apply the `convexed` primitive to the inner envelope which is demonstrated in figure 18.6 and figure 18.7. Of course it is debatable how useful this is but as with all these MetaPost shapes, it has some charm.

²³ Of course adding code is but when looking in more detail at the code involved it was actually possible to simplify the code a bit so there we gained

²⁴ It makes it possible to get points relative to the current point in iterators over paths that we introduced a while ago, which makes for high performance path manipulators.

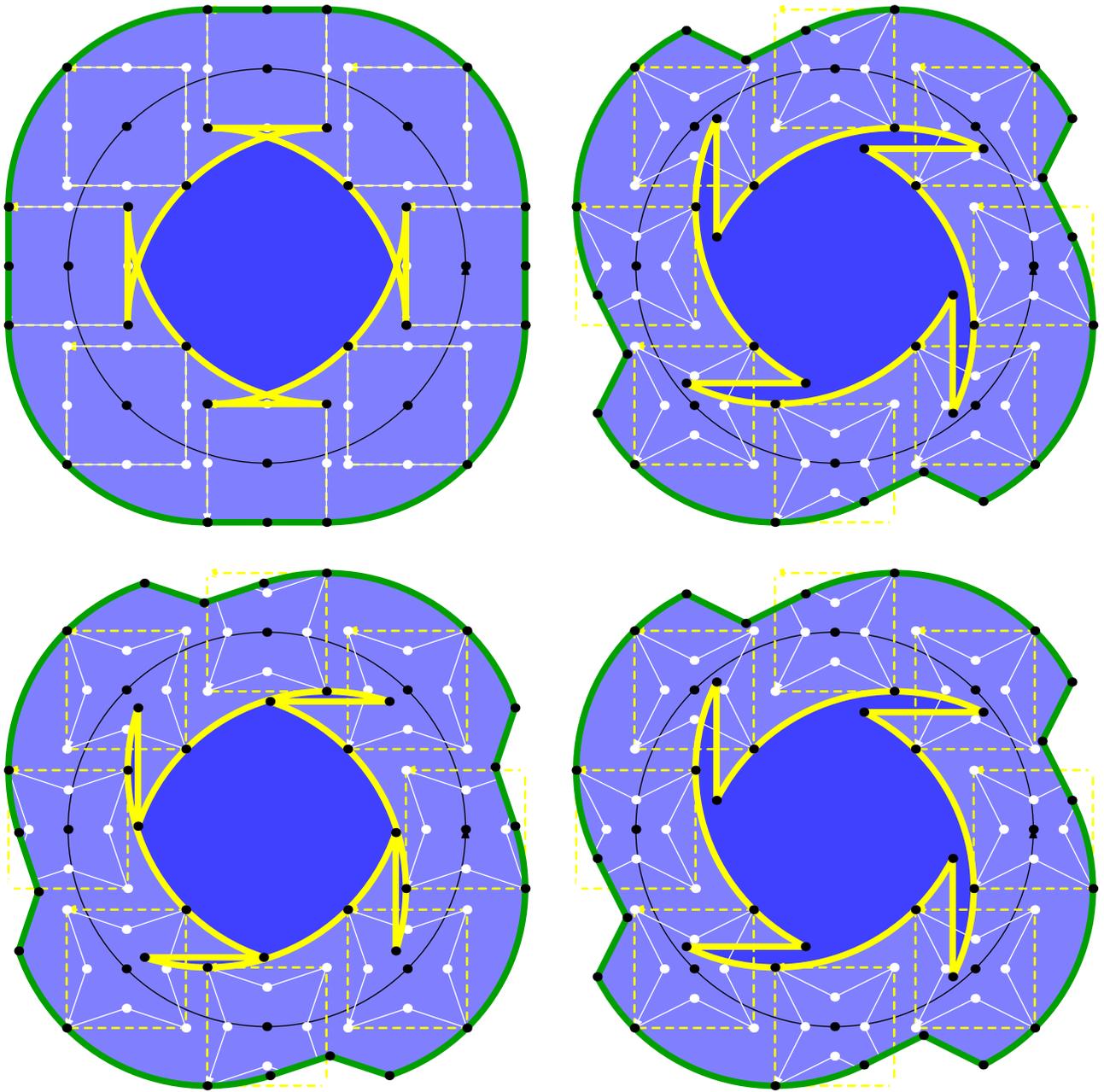


Figure 18.3

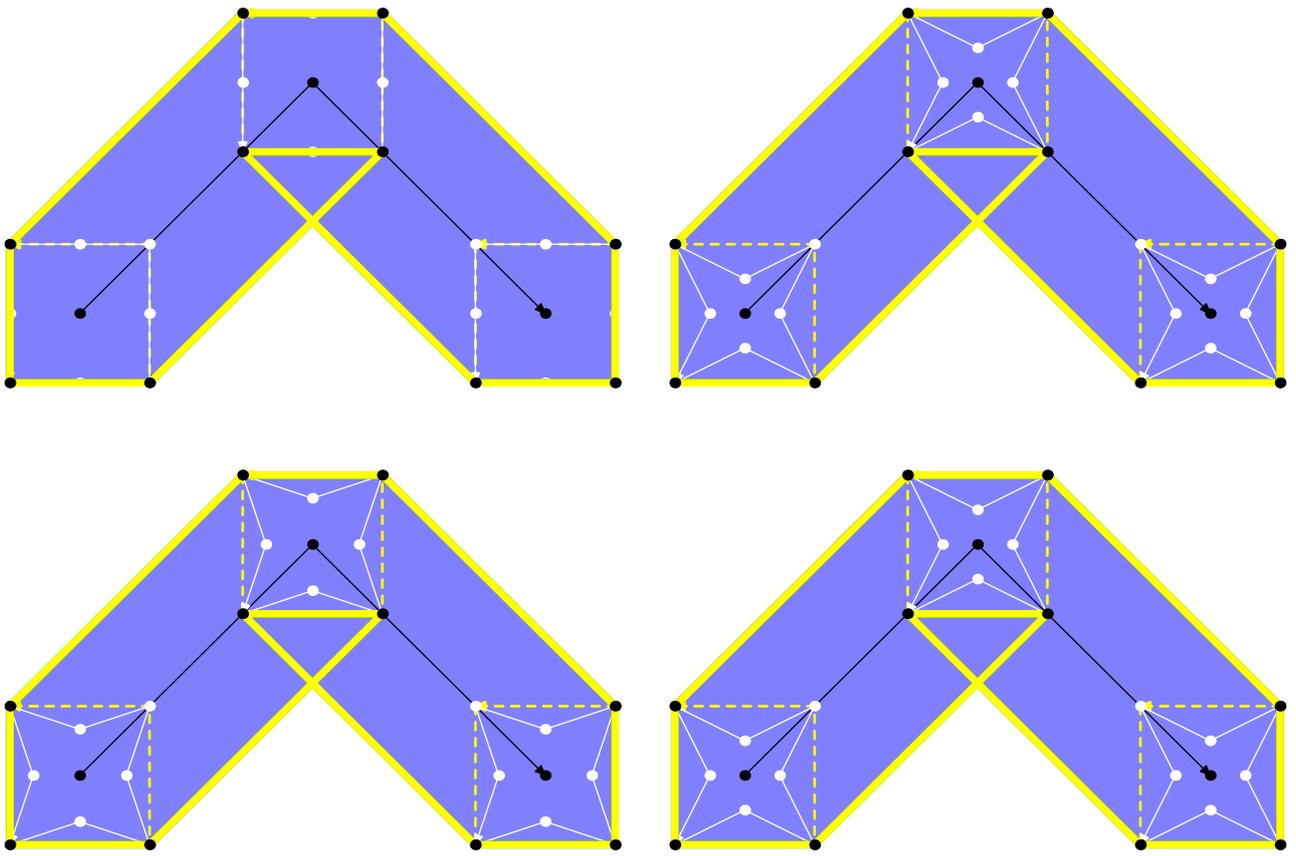


Figure 18.4

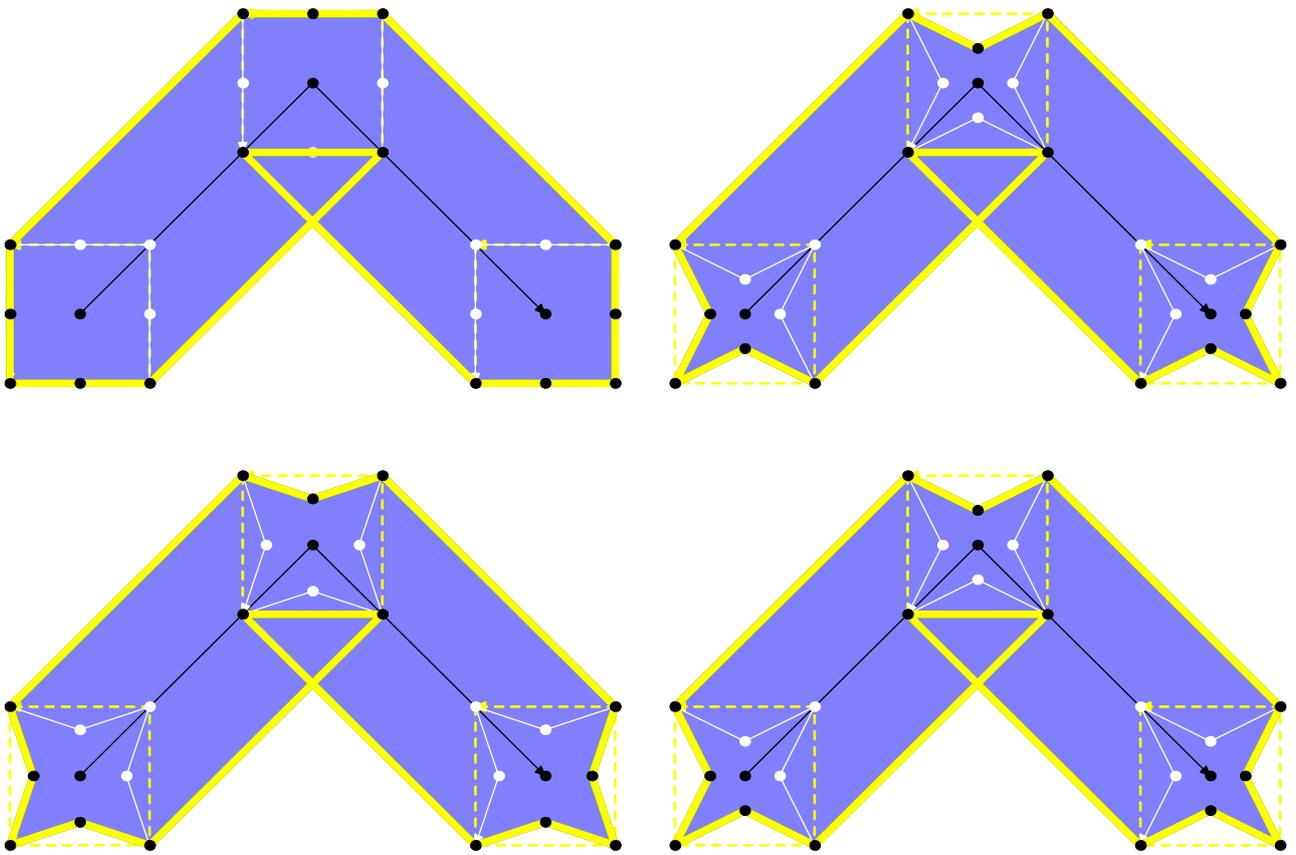


Figure 18.5

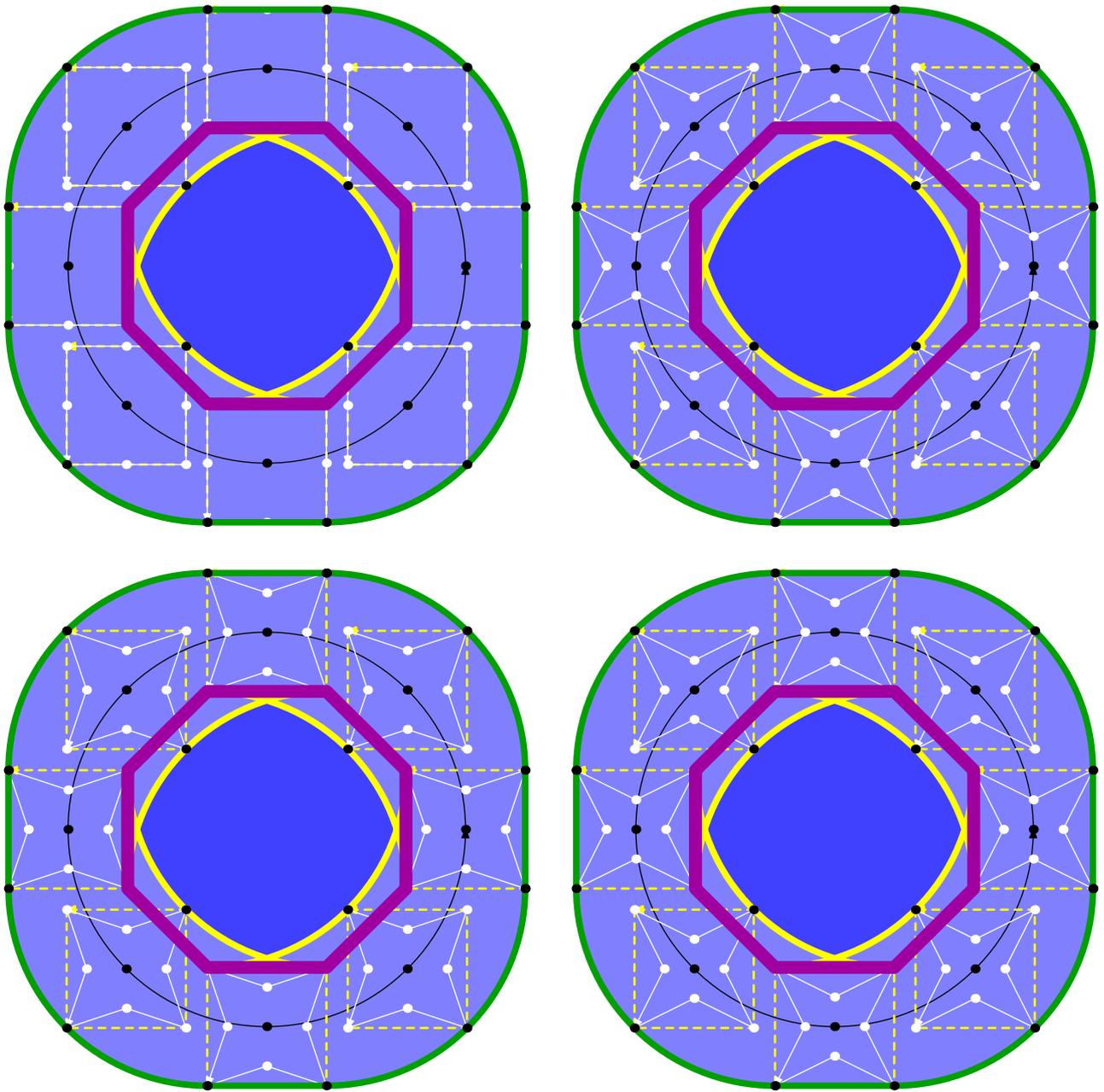


Figure 18.6

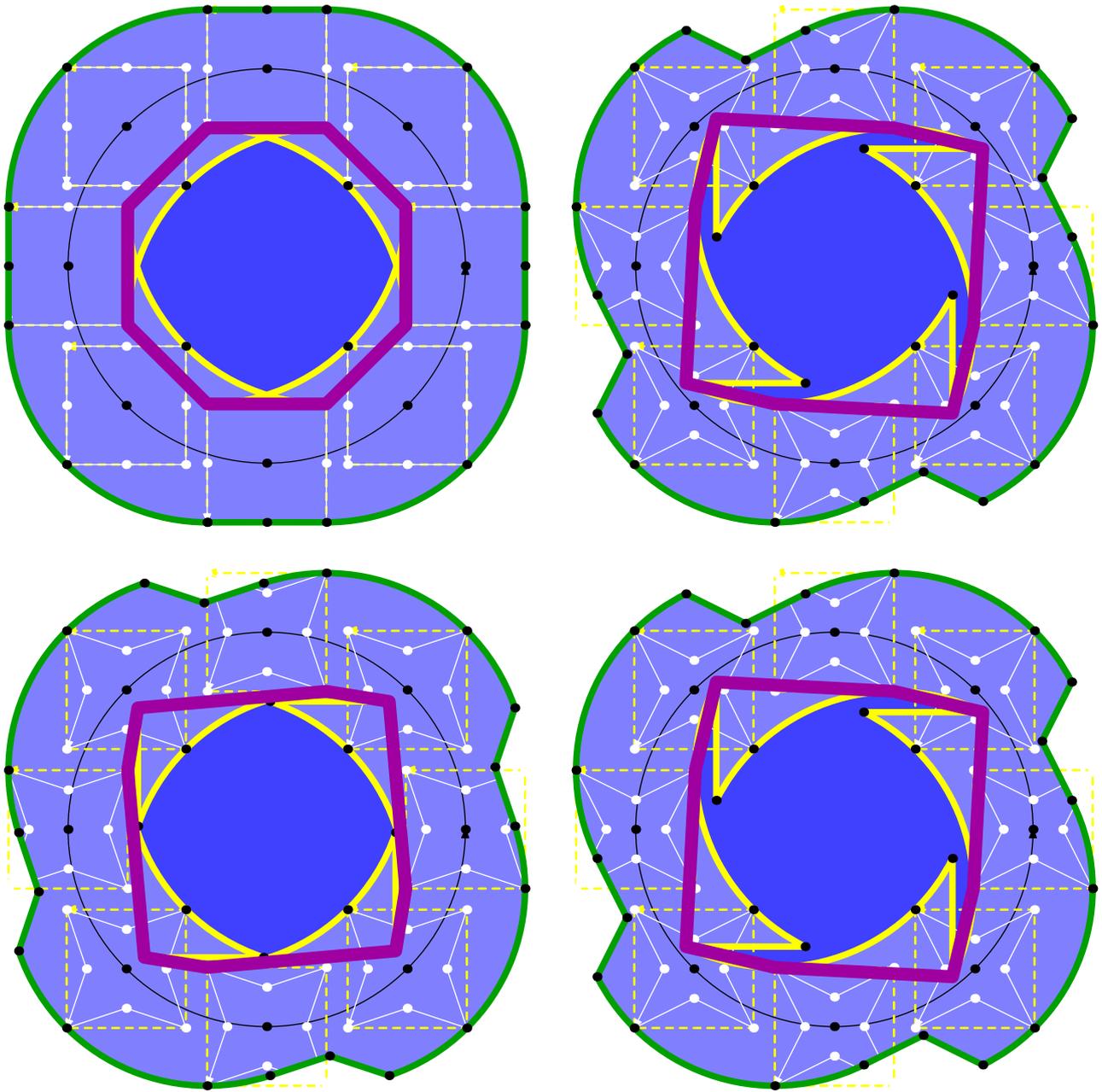
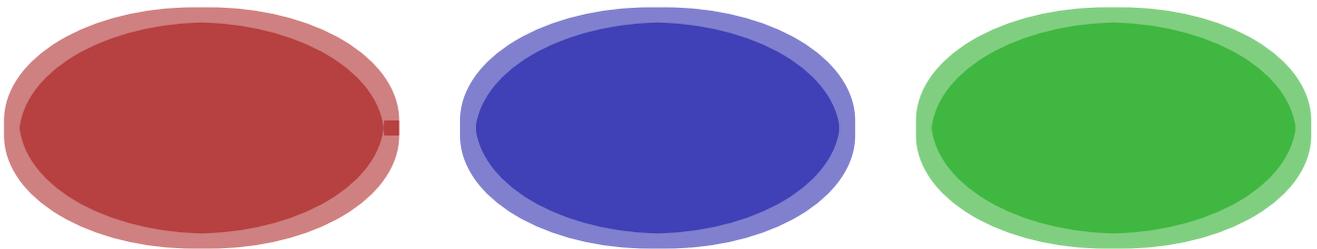


Figure 18.7

To what extent does all this influence the output? As long as we don't use transparencies we're quite okay unless we use a pen size that introduces the more extreme overshoots. If you think these phenomena only relate to MetaPost output, you're wrong. Over the past decades I've seen various fonts that exhibit the same small spikes and other artifacts but as we often see the shapes at small sizes it goes unnoticed. A particular sensitive area is variable fonts where, when the ranges on which the various dimensions operate are too liberal, you can also get these effects. After all, glyphs are filled shapes. To that you can also add the fact that they are single (connected) paths drawn with `eofill`.

The final format a graphics ends up in can be pdf. Take the following three shapes and watch the subtle side effect of rotating either the to be drawn shape or the pen.

```
\startMPcode
  fill fullcircle xyscaled (5cm,3cm)
    withpen makepen(fullsquare) scaled 2mm
    withcolor darkred
    withtransparency (1,.5) ;
  fill fullcircle xyscaled (5cm,3cm)
    shifted (6cm,0)
    withpen makepen(fullsquare rotated eps) scaled 2mm
    withcolor darkblue
    withtransparency (1,.5) ;
  fill fullcircle rotated eps xyscaled (5cm,3cm)
    shifted (12cm,0)
    withpen makepen(fullsquare) scaled 2mm
    withcolor darkgreen
    withtransparency (1,.5) ;
\stopMPcode
```



This produces four filled paths in the pdf file, a normal and a reverse path per shape. I show the whole output because you can see how some points of the 'inside' curve are sort of duplicated: they have the same coordinates but can have different control points.

	inner	outer
left	25=26 31=32 35=36 39=40 43=25	
middle		49=50=51 54=55 58=59 62=63
right	107=108=109 112=113 116=117 120=121	89=105

Here is the output. Each combination is between bound by the transparency operators `/Tr1` and `/Tr0` and has different colors.

```
1 % mps graphic 1: begin
2 q
3 /Tr1 gs
4 0.625 0 0 rg 0.625 0 0 RG
```

5 10 M
6 1 j
7 45.354315 2.83464 m
8 45.354315 14.111559 40.874577 24.926605 32.900591 32.900591 c
9 24.926605 40.874577 14.111559 45.354315 2.83464 45.354315 c
10 2.83464 45.354315 -2.83464 45.354315 -2.83464 45.354315 c
11 -2.83464 45.354315 l
12 -14.111559 45.354315 -24.926605 40.874577 -32.900591 32.900591 c
13 -40.874577 24.926605 -45.354315 14.111559 -45.354315 2.83464 c
14 -45.354315 2.83464 -45.354315 -2.83464 -45.354315 -2.83464 c
15 -45.354315 -2.83464 l
16 -45.354315 -14.111559 -40.874577 -24.926605 -32.900591 -32.900591 c
17 -24.926605 -40.874577 -14.111559 -45.354315 -2.83464 -45.354315 c
18 -2.83464 -45.354315 2.83464 -45.354315 2.83464 -45.354315 c
19 2.83464 -45.354315 l
20 14.111559 -45.354315 24.926605 -40.874577 32.900591 -32.900591 c
21 40.874577 -24.926605 45.354315 -14.111559 45.354315 -2.83464 c
22 45.354315 -2.83464 45.354315 2.83464 45.354315 2.83464 c
23 45.354315 2.83464 l
24 h f
25 39.685035 -2.83464 m
26 39.685035 -2.83464 45.354315 -2.83464 45.354315 -2.83464 c
27 45.354315 -2.83464 45.354315 2.83464 45.354315 2.83464 c
28 45.354315 2.83464 39.685035 2.83464 39.685035 2.83464 c
29 39.685035 -8.442279 35.205297 -19.257325 27.231311 -27.231311 c
30 19.257325 -35.205297 8.442279 -39.685035 -2.83464 -39.685035 c
31 -2.83464 -39.685035 l
32 -2.83464 -39.685035 2.83464 -39.685035 2.83464 -39.685035 c
33 -8.442279 -39.685035 -19.257325 -35.205297 -27.231311 -27.231311 c
34 -35.205297 -19.257325 -39.685035 -8.442279 -39.685035 2.83464 c
35 -39.685035 2.83464 l
36 -39.685035 2.83464 -39.685035 -2.83464 -39.685035 -2.83464 c
37 -39.685035 8.442279 -35.205297 19.257325 -27.231311 27.231311 c
38 -19.257325 35.205297 -8.442279 39.685035 2.83464 39.685035 c
39 2.83464 39.685035 l
40 2.83464 39.685035 -2.83464 39.685035 -2.83464 39.685035 c
41 8.442279 39.685035 19.257325 35.205297 27.231311 27.231311 c
42 35.205297 19.257325 39.685035 8.442279 39.685035 -2.83464 c
43 39.685035 -2.83464 l
44 h f
45 /Tr0 gs
46 0 g 0 G
47 /Tr1 gs
48 0 0 0.625 rg 0 0 0.625 RG
49 158.740139 -2.834616 m
50 158.740139 -2.834252 l
51 158.740139 -2.834252 158.740091 2.835028 158.740091 2.835028 c
52 158.739994 14.111816 154.260267 24.926715 146.286366 32.900615 c
53 138.31238 40.874601 127.497335 45.354339 116.220416 45.354339 c

54 116.220052 45.354339 l
55 116.220052 45.354339 110.550772 45.354291 110.550772 45.354291 c
56 99.273984 45.354194 88.459085 40.874467 80.485185 32.900566 c
57 72.511199 24.92658 68.031461 14.111535 68.031461 2.834616 c
58 68.031461 2.834252 l
59 68.031461 2.834252 68.031509 -2.835028 68.031509 -2.835028 c
60 68.031606 -14.111816 72.511333 -24.926715 80.485234 -32.900615 c
61 88.45922 -40.874601 99.274265 -45.354339 110.551184 -45.354339 c
62 110.551548 -45.354339 l
63 110.551548 -45.354339 116.220828 -45.354291 116.220828 -45.354291 c
64 127.497616 -45.354194 138.312515 -40.874467 146.286415 -32.900566 c
65 154.260401 -24.92658 158.740139 -14.111535 158.740139 -2.834616 c
66 h f
67 153.070811 2.834616 m
68 153.070811 -8.442303 148.591072 -19.257349 140.617086 -27.231335 c
69 132.643186 -35.205235 121.828288 -39.684963 110.5515 -39.685059 c
70 110.5515 -39.685059 116.22078 -39.685011 116.22078 -39.685011 c
71 116.220416 -39.685011 l
72 104.943497 -39.685011 94.128451 -35.205272 86.154465 -27.231286 c
73 78.180565 -19.257386 73.700837 -8.442488 73.700741 2.8343 c
74 73.700741 2.8343 73.700789 -2.83498 73.700789 -2.83498 c
75 73.700789 -2.834616 l
76 73.700789 8.442303 78.180528 19.257349 86.154514 27.231335 c
77 94.128414 35.205235 104.943312 39.684963 116.2201 39.685059 c
78 116.2201 39.685059 110.55082 39.685011 110.55082 39.685011 c
79 110.551184 39.685011 l
80 121.828103 39.685011 132.643149 35.205272 140.617135 27.231286 c
81 148.591035 19.257386 153.070763 8.442488 153.070859 -2.8343 c
82 153.070859 -2.8343 153.070811 2.83498 153.070811 2.83498 c
83 153.070811 2.834616 l
84 h f
85 /Tr0 gs
86 0 g 0 G
87 /Tr1 gs
88 0 0.625 0 rg 0 0.625 0 RG
89 272.125915 2.835004 m
90 272.125819 14.111923 267.645988 24.92693 259.671934 32.900848 c
91 251.697965 40.87468 240.883028 45.354315 229.606241 45.354315 c
92 229.606241 45.354315 223.936961 45.354315 223.936961 45.354315 c
93 223.936596 45.354315 l
94 212.659677 45.354219 201.84467 40.874388 193.870752 32.900334 c
95 185.89692 24.926365 181.417285 14.111428 181.417285 2.834641 c
96 181.417285 2.834641 181.417285 -2.834639 181.417285 -2.834639 c
97 181.417285 -2.835004 l
98 181.417381 -14.111923 185.897212 -24.92693 193.871266 -32.900848 c
99 201.845235 -40.87468 212.660172 -45.354315 223.936959 -45.354315 c
100 223.936959 -45.354315 229.606239 -45.354315 229.606239 -45.354315 c
101 229.606604 -45.354315 l
102 240.883523 -45.354219 251.69853 -40.874388 259.672448 -32.900334 c

```

103 267.64628 -24.926365 272.125915 -14.111428 272.125915 -2.834641 c
104 272.125915 -2.834641 272.125915 2.834639 272.125915 2.834639 c
105 272.125915 2.835004 l
106 h f
107 266.456635 -2.834276 m
108 266.456635 -2.834641 l
109 266.456635 -2.834641 266.456635 2.834639 266.456635 2.834639 c
110 266.456635 -8.442148 261.977 -19.257085 254.003168 -27.231054 c
111 246.02925 -35.205108 235.214243 -39.684939 223.937324 -39.685035 c
112 223.936959 -39.685035 l
113 223.936959 -39.685035 229.606239 -39.685035 229.606239 -39.685035 c
114 218.329452 -39.685035 207.514515 -35.2054 199.540546 -27.231568 c
115 191.566492 -19.25765 187.086661 -8.442643 187.086565 2.834276 c
116 187.086565 2.834641 l
117 187.086565 2.834641 187.086565 -2.834639 187.086565 -2.834639 c
118 187.086565 8.442148 191.5662 19.257085 199.540032 27.231054 c
119 207.51395 35.205108 218.328957 39.684939 229.605876 39.685035 c
120 229.606241 39.685035 l
121 229.606241 39.685035 223.936961 39.685035 223.936961 39.685035 c
122 235.213748 39.685035 246.028685 35.2054 254.002654 27.231568 c
123 261.976708 19.25765 266.456539 8.442643 266.456635 -2.834276 c
124 h f
125 /Tr0 gs
126 0 g 0 G
127 Q
128 % mps graphic 1: end

```

The duplicates differ per variant and as they are effective `lineto` combine with `curveto` we can consider removing the `lineto`'s. This can either be done in the backend or we can decide to do that in the MetaPost library during the export.²⁵

The `tracingspecs` flag can help us to see what happens deep down when envelopes are made. It will show the intermediate path on the console.

```

tracingspecs := 1;
path e ; e := envelope (makepen(fullsquare) scaled 2mm) of (fullcircle scaled 3cm)
;
show(e);

```

The intermediate path is reported as:

```

% beginning with      offset ( 2.83464, 2.83464)
( 42.51968, 0          ) .. controls ( 42.51968, 11.27742) and ( 38.03908,
                                                                22.09160)
.. ( 30.06534, 30.06534) .. controls ( 22.09160, 38.03908) and ( 11.27742,
                                                                42.51968)
% counterclockwise to offset (-2.83464, 2.83464)

```

²⁵ In the end I settled on introducing a move tolerance in addition to the bend tolerance that we already have in the export. The default value of 0.001999 removes 14 lines from the above pdf code.

```

.. ( 0, 42.51968) .. controls ( 0, 42.51968) and ( 0,
42.51968)
.. ( 0, 42.51968) .. controls (-11.27742, 42.51968) and (-22.09160,
38.03908)
.. (-30.06534, 30.06534) ..controls (-38.03908, 22.09160) and (-42.51968, 11.27742)
% counterclockwise to offset (-2.83464,-2.83464)
.. (-42.51968, 0 ) .. controls (-42.51968, 0 ) and (-42.51968, 0
)
.. (-42.51968, 0 ) .. controls (-42.51968,-11.27742) and
(-38.03908,-22.09160)
.. (-30.06534,-30.06534) .. controls (-22.0916, -38.03908) and
(-11.27742,-42.51968)
% counterclockwise to offset ( 2.83464,-2.83464)
.. ( 0, -42.51968) .. controls ( 0, -42.51968) and ( 0,
-42.51968)
.. ( 0, -42.51968) .. controls ( 11.27742,-42.51968) and ( 22.0916,
-38.03908)
.. ( 30.06534,-30.06534) .. controls ( 38.03908,-22.09160) and (
42.51968,-11.27742)
% counterclockwise to offset ( 2.83464, 2.83464)
.. ( 42.51968, 0 ) .. controls ( 42.51968, 0 ) and ( 42.51968, 0
)
.. ( 42.51968, 0 )
& cycle

```

The result becomes:

```

( 45.35432, 2.83464) .. controls ( 45.35432, 14.11206) and ( 40.87372,
24.92624)
.. ( 32.89998, 32.89998) .. controls ( 24.92624, 40.87372) and ( 14.11206,
45.35432)
.. ( 2.83464, 45.35432) .. controls ( 2.83464, 45.35432) and ( -2.83464,
45.35432)
.. ( -2.83464, 45.35432) .. controls ( -2.83464, 45.35432) and ( -2.83464,
45.35432)
.. ( -2.83464, 45.35432) .. controls (-14.11206, 45.35432) and (-24.92624,
40.87372)
.. (-32.89998, 32.89998) .. controls (-40.87372, 24.92624) and (-45.35432,
14.11206)
.. (-45.35432, 2.83464) .. controls (-45.35432, 2.83464) and (-45.35432,
-2.83464)
.. (-45.35432, -2.83464) .. controls (-45.35432, -2.83464) and (-45.35432,
-2.83464)
.. (-45.35432, -2.83464) .. controls (-45.35432,-14.11206) and
(-40.87372,-24.92624)
.. (-32.89998,-32.89998) .. controls (-24.92624,-40.87372) and
(-14.11206,-45.35432)
.. ( -2.83464,-45.35432) .. controls ( -2.83464,-45.35432) and (
2.83464,-45.35432)

```

```

.. ( 2.83464,-45.35432) .. controls ( 2.83464,-45.35432) and (
                                         2.83464,-45.35432)
.. ( 2.83464,-45.35432) .. controls ( 14.11206,-45.35432) and (
                                         24.92624,-40.87372)
.. ( 32.89998,-32.89998) .. controls ( 40.87372,-24.92624) and (
                                         45.35432,-14.11206)
.. ( 45.35432, -2.83464) .. controls ( 45.35432, -2.83464) and ( 45.35432,
                                         2.83464)
.. ( 45.35432, 2.83464) .. controls ( 45.35432, 2.83464) and ( 45.35432,
                                         2.83464)
.. cycle

```

Numerous experiments by Mikael and me lead to the conclusion that both stages can introduce the duplicate points and that any messing with that during envelop generation time has negative side effects. However, when we export the path we can definitely get rid of them. They are harmless but we're talking quality control here and \TeX and MetaPost is all about quality!

As usual, playing with mechanisms like this gets one wondering about similar cases, for instance variants of dashing.

```

\startMPcode
vardef dashing (expr pth, shp, stp) =
  for i within arcpointlist stp of pth :
    shp
      rotated angle(pathdirection)
      shifted pathpoint
    &&
  endfor nocycle
enddef ;

path parrA ; parrA :=
  (0,0) -- (0,-1) -- (2,-1) -- (2,-2) -- (4,0) -- (2,2) -- (2,1) -- (0,1) --
                                         (0,0)
;
path parrB ; parrB :=
  parrA -- (0,-1) -- (2,-1) -- (2,-2) -- (4,0)
;
path p ; p := fullcircle scaled 2cm ;

fill (dashing (p, parrA, 25) && cycle) withtransparency (1,.5) ;
draw (dashing (p, parrA, 25) && cycle) withtransparency (1,.5) ;
fill (dashing (p, parrB, 25) && cycle) shifted (3cm,0) withtransparency (1,.5) ;
draw (dashing (p, parrB, 25) && cycle) shifted (3cm,0) withtransparency (1,.5) ;
\stopMPcode

```

In figure 18.8 we see the result. Of course how well it comes out depends on the definition but what is special here is that we use the double ampersand operator. That one will connect the paths without complaining about the end and being point not colliding. I suppose there was a good reason for making that a condition in the case of fonts, after all MetaFont is what it came from, but there is no real reason for it. It is a cheap extension anyway. At the same time I decided to add a native 'direction' operator. The number of extra bytes

in the binary is probably less than what is needed in memory to store the macro and the advantage is that we save an extra run over the path to reach the point we're consulting.²⁶

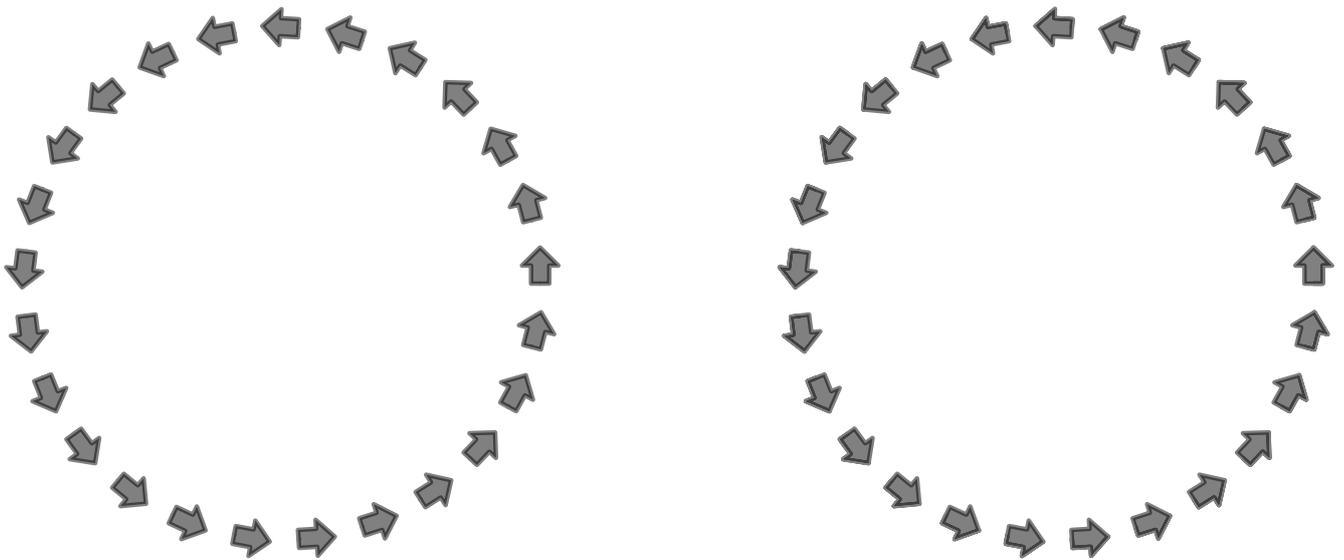


Figure 18.8 A somewhat related rendering.

In case you wonder why we need this feature, here is an argument:

```
\startMPcode
  path s ; s := fullcircle scaled 4cm ; pickup pencircle scaled 5mm ;

  draw (s shifted (0cm,0) && s shifted (3cm,0) && s shifted (6cm,0))
    withcolor "darkred" withtransparency (1,.5) ;

  currentpicture := currentpicture shifted (-8cm,0) ;

  draw s shifted (0cm,0)
    withcolor "darkblue" withtransparency (1,.5) ;
  draw s shifted (3cm,0)
    withcolor "darkblue" withtransparency (1,.5) ;
  draw s shifted (6cm,0)
    withcolor "darkblue" withtransparency (1,.5) ;

  currentpicture := currentpicture shifted (-8cm,0) ;

  nodraw s shifted (0cm,0) ;
  nodraw s shifted (3cm,0) ;
  nodraw s shifted (6cm,0) ;
  dodraw origin withcolor "darkgreen" withtransparency (1,.5) ;

\stopMPcode
```

The results are shown in figure ???. Which if the alternatives you prefer also depends on how you generate the shape. The `nodraw` variant for instance can be mixed with calculations without the need to revert to `hide`.

²⁶ In case you wonder, this is how the macro definition looks like: `vardef direction expr t of p = postcontrol t of p - precontrol t of p enddef ;`. Because points are searched from from the start there are two lookups needed. Normally this is no problem but Mikael and I are playing with really large paths, like those that come from drawing functions.

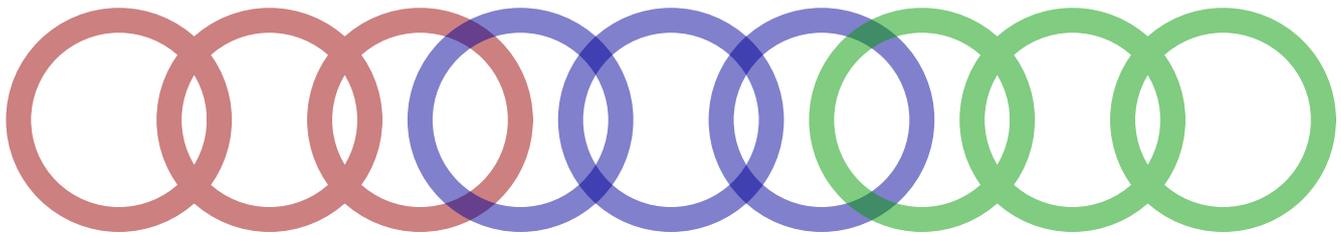


Figure 18.9 Do you see the difference?

Figure 18.10 demonstrates how far we've come. Mikael's fancy arrows nicely follow the shape of the function. Of course you need to make sure that these arrows are reasonably scaled. The definition of `dashing` demonstrates a few primitives that permits efficient iteration over a path and `arcpointlist` is sort of a path.

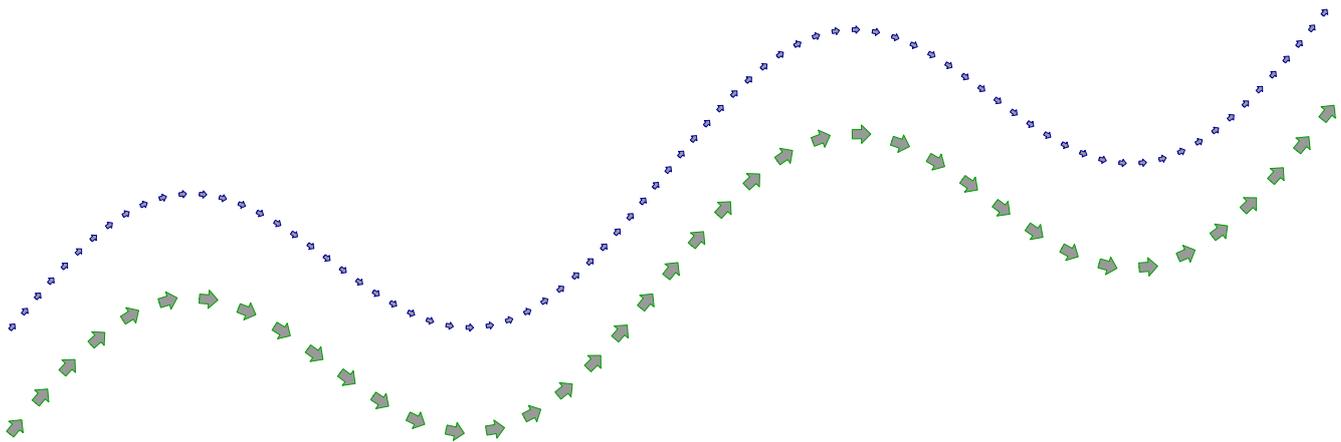


Figure 18.10 Advanced pseudo dashing.

```
\startMPcode
vardef dashing (expr pth, shp, stp) =
  for i within arcpointlist stp of pth :
    shp
      rotated angle(pathdirection)
      shifted pathpoint
    &&
  endfor nocycle
enddef ;

path e, p ; numeric n ;
e := (0,0) -- (0,-1) -- (2,-1) -- (2,-2) -- (4,0) -- (2,2) -- (2,1) -- (0,1) --
(0,0) ;

n := 10 * bbwidth(e) ;
p := function(1,"x","x/4 + sin(x)",epsed(0.1),epsed(4*pi),0.01) scaled 2cm ;

fill (dashing (p, e scaled 2.5, n) && cycle) withcolor .6white ;
draw (dashing (p, e scaled 2.5, n) && cycle) withcolor darkgreen ;

currentpicture := currentpicture shifted (0,-2cm) ;

n := 20 * bbwidth(e) ;
fill (dashing (p, e, n) && cycle) withcolor .6white ;
draw (dashing (p, e, n) && cycle) withcolor darkblue ;
\stopMPcode
```

19 Dealing with math fonts

Introduction

Here we will explain some of the tricks that we apply to math fonts so that they not only work better with the LuaMetaT_EX math engine but also look better, at least in our opinion. We will not show specific fonts because after all, who can complain about something that comes for free, but you can see whatever we do to make it work in action in ConT_EXt where we setup these fonts. This is also a summary of what Mikael Sundqvist and I have been doing for a while now: improve the rendering of math, a rather enjoyable experience, also because we ran into humorous effects with and properties of fonts. Because we consider ourselves free from any conventions we could happily explore solution.

Fences

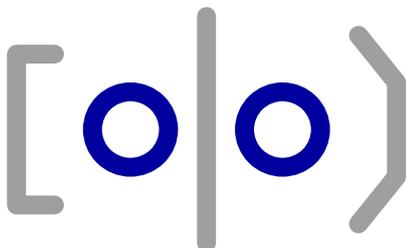
Fences come in two variants: fixed sizes and so called extensibles that are constructed from recipes that combine snippets that can partially overlap. Fenced material has an optional symbol at the start, an optional one at the end and zero or more symbols in the middle. Here we have all three:



Ideally these symbols scale in the same way, depending on the other context. This means that T_EX first has to measure what sits in between but we will not dive into that. The left and right symbols are normally pairs like parentheses or braces, but any mix is possible. Ideally a font is designed with this in mind but unfortunately we see this:



And even this:

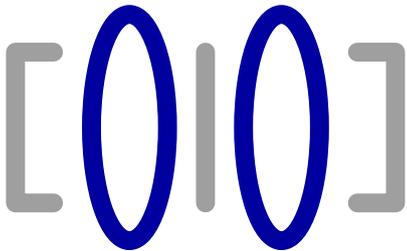


It might be a side effect of the limited amount of available slots in traditional math fonts that also resulted in non consistent sets in OpenType follow-ups and when one font does that more follow that approach.

You can find rendering like this:

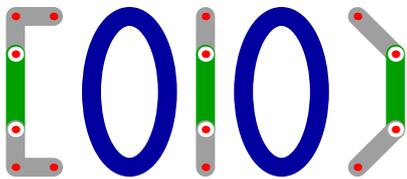


and this

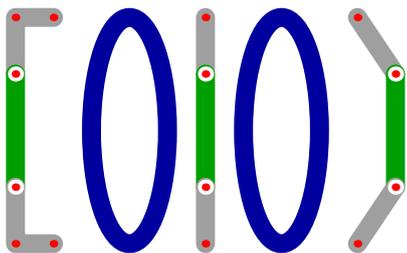


because a programmable language like \TeX can use some tricks to force sizes: we just create some local fence which dimension is determined by some invisible rule. In $\text{LuaMeta}\TeX$ we can actually enforce dimensions and in $\text{Con}\TeX$ t we can filter specific sizes.

So how does this sizing work? A fence character starts out with the normal size but then a larger one is needed, the math engine will check if there is a larger variant. An OpenType font can provide these and in the engine that works out as following a linked list to a next size. When we run out of sizes there can be an extension recipe present where a fence is made from snippets pasted together. Normally that goes unnoticed because there is a little overlap between these snippets.



A larger fence will simply add more middle pieces, and it will not do as below:



Because we're talking of a deliberate design you cannot simply scale snippets and expect them to work out well visually. However, in a pure vertical case one actually could and in practice all these extensibles have (of course) vertical bars. Anyway, in the above example the larger middle piece actually is just several middle pieces overlapping.

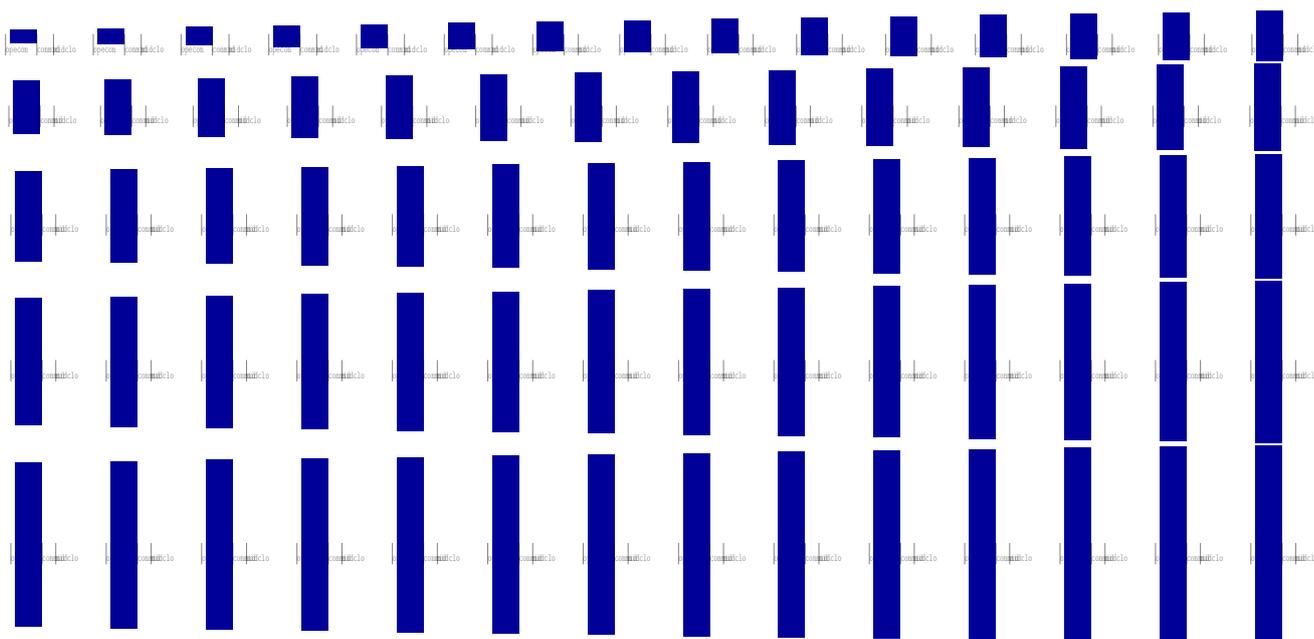
However, as we mentioned, fonts are not always consistent. First of all, when we run over the (increasing in size) variants we have discrete steps and you're lucky if a font has more than half a dozen. As soon as we end up with the extensible the size can be matched well.

So how do we compensate for misbehavior? There are two parameters in \TeX then determine the matching: `\delimiterfactor` and `\delimitershortfall`. Plain \TeX set them to 901 and 5pt which works okay in

most cases. In ConTeXt we set them to 1000 and Opt and instead use the parameters `\UmathDelimiterPercent` and `\UmathDelimiterShortfall` that are bound to fonts. In addition to that we use the `nooverflow` keyword with `\Umiddle` which makes sure that we always stay within the size of the outer fences. That just looks better.

In addition we can tweak the dimensions of glyphs and apply effects such as expanding so that we get a bit more consistent visual appearance. We can also signal that we should ignore sizes larger than a given index.

The next sequence show what happens in practice when we tell `\Umiddle` to never exceed the requested size. Because we start with stepwise sizes the first part of this sequence has no matching sizes. At some point we end up at the extensibles.



It is worth noticing that we tried several alternative approaches. For instance what happens when we only use extensibles? In that case there will be no fit for the smaller ones because the at least two parts of an extensible can seldom completely overlap that much.

Actually, when we tested that we noticed that even in valid situations there can be strange overlap. At that time for instance the Lucida fonts had overlapping artifact in some curly braces which we found out when we tried to nil some of the larger, odd looking, step variants. Latin modern and some of the gyre fonts had unexpected jumps to larger sizes which made us decide to make the delimiter parameters font specific so that we could more easily adapt them: they basically became part of the math parameters of a font.

There are also inconsistencies in the perceived widths of glyphs used: often the bars are too thin. That can be solved by applying effects like scaling horizontally or vertically and cheating a bit with the dimensions. Another solution is that we ignore the variants after a certain size and force extensibles sooner but that of course needs to be tested for unwanted overlaps too. All these tricks combined make it possible to use math fonts with imperfect fences more or less reliable.

- Provide an equal amount of fixed size larger variants for all fences: assume arbitrary pairing.
- Because fonts have plenty room, provide some ten variants before going extensible.
- Try to make the variants and the extensibles similar in look.
- Ensure that the width of the vertical bar matches the design.
- Make sure that the odd entries in a extensible recipe don't overlap badly.

Accents

When traditional \TeX showed up it was not that common to have pre-composed characters so when you needed something with an accent on top the way to go was to typeset the base character and position the accent on top using either the `\accent` primitive or some macro. It always was a compromise but eventually fonts with more assembled characters showed up. In OpenType fonts that operate in the Unicode domain we have even more characters but even there characters can be composed. However, anchors that help achieving this are part of the format. For text we use the mark features and for math we use the top anchor. Given that, why do we need to tweak it?

Here we have a base character with an accent on top. The character is upright and the accent gets positioned in the middle.



This doesn't work out well if we have a slanted or italic shape:



So we need to compensate, for instance like this:

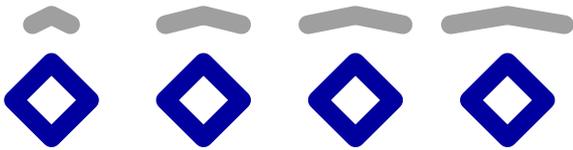


However, what does determine the right anchor point? From this example you can conclude that it is the top of the character. It is probably for that reason why the semi automated construction of Latin Modern and the Gyre fonts have quite some anchors that are rather bad: getting the anchors right is more a visual job than something that can be automated. The topmost point is not really the best one to focus on.

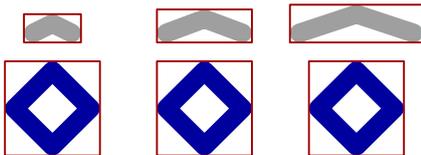


Here the topmost position is very off center. In for instance Latin Modern that means that on digits like 7 and 4 you get very weird anchoring. And this is why we have a tweak that just wipes all the anchors from an alphabet: most alphabets don't need them anyway and the engine will use the center when no anchor is defined. Just for the record: in traditional \TeX engines the horizontal position is determined by the kern between a so called skew character and the base character. The font format has no anchor field but it has kerns, so this trick makes much sense.

We discussed vertical extensible that grow but horizontally we have accents that can grow. There are also a few horizontal fences like braces that have extensibles but we will cover that later. Accents are such that they only have a fixed set of variants and one problem is that there are often not enough of them. This means that the engine has to choose one that is reasonable.



In the example above the first two are acceptable but the third and fourth are not. Just imagine that there is a superscript or subscript involved. Here we apply another cheat: we lie about the dimensions. A glyph can have left and right margins that get subtracted when the accent analyzer tries to make a fit which means that we can sort of enforce the second solution. The ConT_EXt font goodie files set the margins for some problematic characters because (of course) these are not part of OpenType math fonts. This is an optical issue mostly because the engine will not easily put a too wide one on top.



Watch how the larger accents also have a larger bounding box. That is all right but does interfere with a consistent makeup. The solution is simple: we use the ConT_EXt dimension tweak to reposition accents and cheat with their height and depth to make sure that we get consistent rendering. We need to tweak anyway because sometimes accents have bad dimensions. The smallest one is actually a text accent and therefore can have properties that are inconsistent with its wider variants. This is typically a side effect of the fact that math accents and text accents are not considered to be different.

- Only add anchors to some (forward leaning!) italic shapes.
- Make extensibles as much a possible consistent with respect to dimensions.

Kerning

In a text font there are two mechanisms that influence the spacing between individual characters: kerning and italic corrections. In OpenType text fonts we have a more generalized relative positioning mechanism which can be seen as kerning. The italic correction well known to T_EX users can be implemented as a positional font feature but very seldom is.

An OpenType math font has both kerning and italic correction. The kerning at the left top, left bottom, right top and right bottom of a glyph can be specified as a staircase and is used to position scripts. The italic correction is bit more curious and is applied in some cases. Keep in mind that in math a sequence of alphabetic characters does not make a word but represents a multiplication of ordinary symbols and thereby specific inter-atom spacing rules apply.

A traditional T_EX font has kerns and italic correction and an OpenType font has staircase kerns and italic correction. For practical (space and time) reasons the widths of italic shapes in traditional math fonts are such that when you add the correction they kind of match the bounding box.

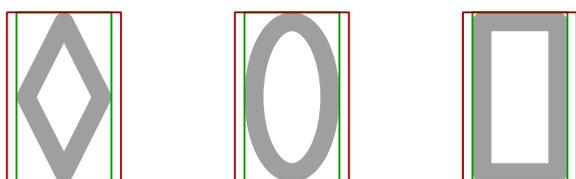
So, one way to suit both kind of fonts is to add the italic correction (often absent in OpenType glyphs but very present in traditional ones) as well as the staircase kerns (present in OpenType fonts and unknown to traditional fonts).

There are however some complications: what if a glyph has both? Which one is to be preferred? Should we try our luck? Even worse: in the traditional the italic correction is *always* added to the box that wraps a

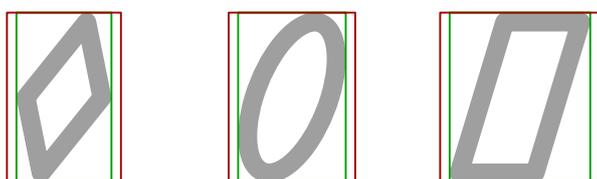
glyph but in some cases that correction gets removed.²⁷ But should we also remove a staircase kern? When we started with LuaMetaTeX it was a bit of a gamble because the specification only showed up later and improved over time.²⁸

In LuaTeX we often have to code paths. That was done after other attempts to deal with this weak aspect of fonts worked for one font and not for the other. For instance at the time of this writing some fonts have italic corrections for upright characters! In LuaMetaTeX that model was changed into a detailed control model at the font as well as engine level. Later, when Mikael and I went over the fonts, usage of characters in math, atoms and spacing, we decided to kick out that detailed control and use more general control mechanisms assuming that we use OpenType fonts without bad italics. Whenever we had bad ones we could correct that in a so called font goodie file. In other words: no heuristics in the engine but fixed fonts. For that we always take Cambria as reference.

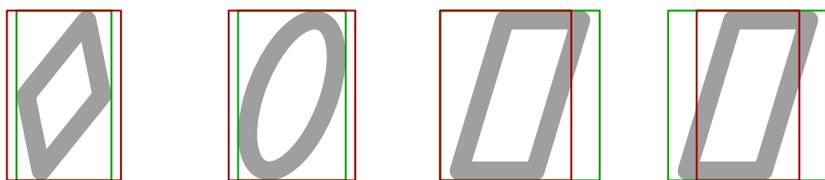
To be considered: should we finally turn italic correction into top and bottom kerns? Basically: model after Cambria? Then we can kick out code! We just assume font goodies.



Here we have three shapes and as usual they have some space at the left and right. A text font like Lucida is designed in such a way that no kerns between glyphs are needed but most text fonts have kerns. These compensate for these average acceptable side bearings.



In these slanted versions we get wider shapes but not always. For some shapes the amount of perceived spacing at the left top and right bottom increases and this is where we start thinking in terms of italic correction:



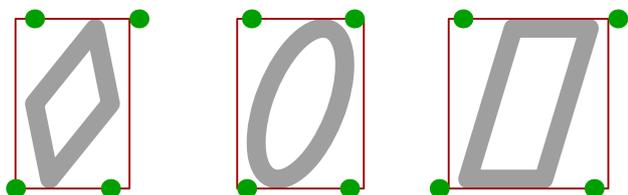
That way, when we put characters next to each other on the average words look better. But how about math: a superscript should be outside that bounding box and a subscript inside, assuming that we have a shape like this. It is unfortunate that the widely used $\frac{1}{2}$ is the perfect candidate. When using that one as test case things can look great but kick in a $\frac{1}{2}$ and it gets worse. The $\frac{1}{2}$ and $\frac{1}{2}$ are also a fertile playground. It is hard to come up with logic that satisfied all which is why glyph specific engine control was implemented (and later dropped). In the previous graphic the fourth shape also cheats on the left and yes, there are some fonts that do just that, which of course interferes with prescripts.

²⁷ In LuaMetaTeX we never remove but compensate so that we can track what happens.

²⁸ This is true for much of OpenType which has the danger that bugs and side effects become features. Keep that in mind when you criticize solutions that early adopters came up with!

Now think of using shapes in formulas: some are put in sequence, in which case inter atom spacing is added so there is less danger of touching due to too a narrow width and $\text{T}_{\text{E}}\text{X}$ ies somehow accepted that adding thin spaces every now and then is fine²⁹ But there's more than sequences: shapes end up in scripts, as degrees in radicals, above and below fraction rules, as fences and accents. Just assume the worst possible scenarios!

In these examples the italic correction at the right is the difference between the red and green box. There is no left italic correction and that is why OpenType math (driven by Cambria) has these four sets of kerns.



Here we show some possibilities for better anchoring but it will be clear that it is a compromise. Staircase kerns as in OpenType therefore have a set of kerns going up or down for not only the base character but also the one that ends up in a script as that one itself can have a 'problematic' shape.

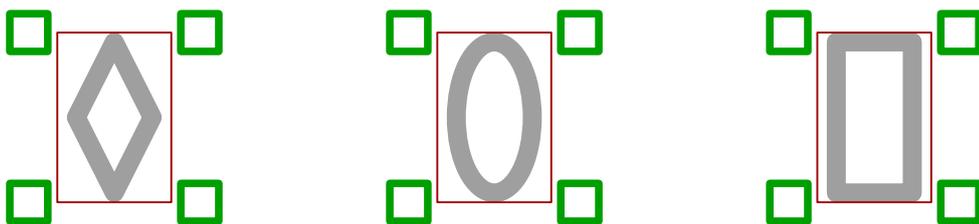
Our solution to this problem is to tweak dimensions and italic correction of problematic characters. We also can add four kerns that roughly compensate for tricky corrections needed. We therefore have more glyph properties than the official OpenType specification provides (cheaper and easier than staircase kerns that work on pairs of characters). But we have more font parameters anyway, and with $\text{T}_{\text{E}}\text{X}$ being one of the main math renderers and $\text{T}_{\text{E}}\text{X}$ ies always being to tune and tweak we think this is okay. It is better to have more control than to rely on (hard to fight) heuristics.

We're stuck with the fonts we have and (in the case of $\text{T}_{\text{E}}\text{X}$ fonts) the chosen traditional approach of dimensions and italic corrections (no staircase kerns) but the least we can ask is:

- Be consistent in dimensions and italic corrections. Get rid of limitations imposed by the 8 bit font era: we have plenty slots available and some more glyph properties as well. And if you compromise: make that clear.
- Don't just take the old properties but assume that OpenType math fonts are used in a modern OpenType $\text{T}_{\text{E}}\text{X}$ engine.
- Assume that any character is used in any combination: that is what made it hard to satisfy all needs at the engine end. Better play safe.

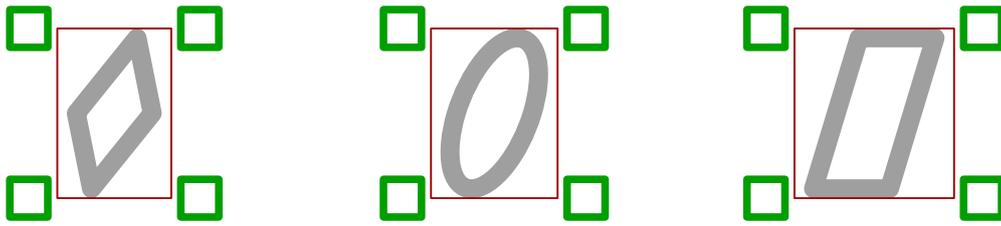
Scripts and primes

We start with showing a few shapes (in $\text{T}_{\text{E}}\text{X}$ speak: nuclei) with a different perceived spacing and with some items attached to the corners: scripts.

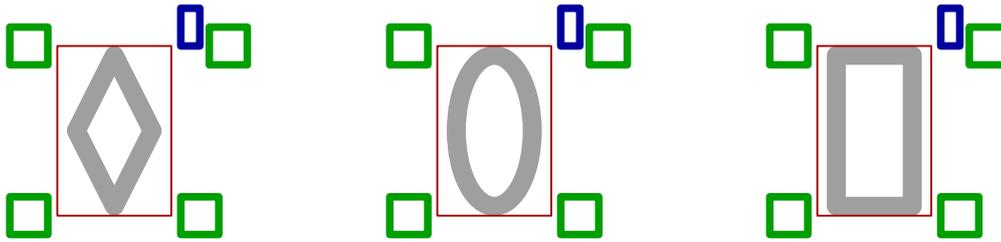


The green ones represent the super- and sub-, pre- and postscripts. According to what we discussed previously the anchoring depends on the shape

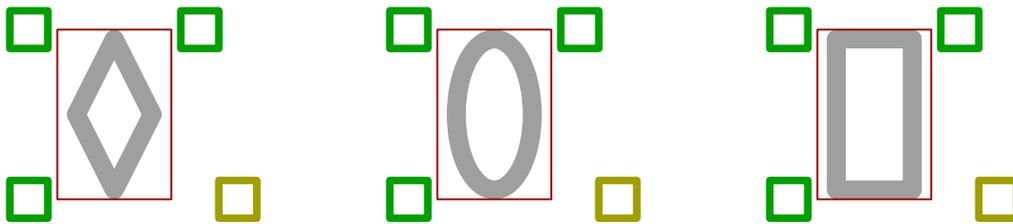
²⁹ We don't think so which is why we came up with a more granular inter atom spacing mode.



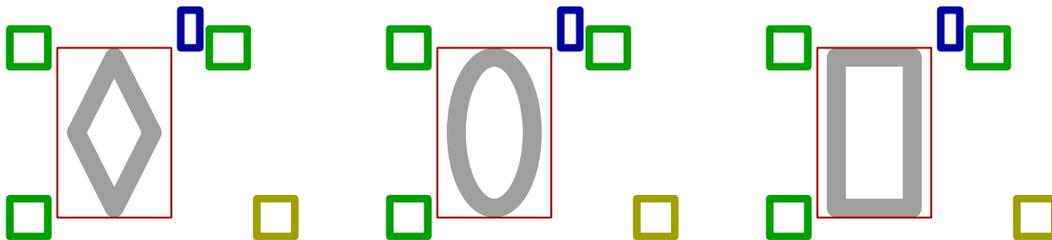
But we will not take that into account here because this time we focus on support for primes and indices. A prime normally is a narrow character that is positioned after the nuclues and sits in a similar position as a superscript. When it is present a subscript doesn't move. Here the blue rectangle represents the prime.



An index, here yellow, is a subscript that applies to the whole so that one does move. We cannot have a subscript and an index at the same time. The prescripts are not affected by primes and indices other than that the engine has to do more work in getting it right.



Of course we can have primes and indices at the same time. A prime actually belongs to the nuclues so that combination determines quite a bit of the vertical and horizontal spacing.



The problem with primes is that they were never part of the concept and OpenType math inherited that. Thanks to Unicode (and the math community not being convincing enough) we ended up with a prime being equivalent to minutes and double primes being minutes. The fact that we have triple and quadruple primes as well as reverse primes is a consequence of not having symbols for sub second indicators. And even the seconds are overloaded by meaning: time related and geographical.

What has this to do with primes? Well, it means that a font has a prime character that is already positioned at a certain distance from the baseline. But in most math fonts the script and scriptscript sizes aren't. It looks as if the assumption is that primes are treated like superscripts. But which one then gets superscripted at the text level? The text one or the script one? Unfortunately fonts are somewhat inconsistent in the sizing and positioning of primes and the math community not being convincing enough. It was a tough decision to make: in some fonts using the text prime have nice output and in other fonts the script one.

So, because in the traditional approach primes had to be manually positioned, be able deal with super and subscripts the old school approach was to make them active characters and pick up what follows in order to deal with this situation. It probably is the main reason why we have a somewhat special active character mechanism in math mode. In ConT_EXt MkIV we followed a different approach, also because we wanted to deal with collapsing multiple primes to their rightful Unicode slot.

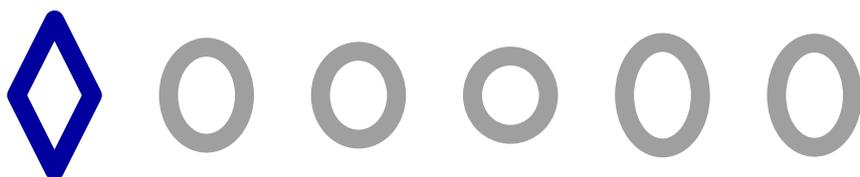
In LMTX and LuaMetaT_EX we (need to) go a step further because we want proper inter-atom space as well as more fine tuned positioning. Primes became elements bound to a nucleus! We also wanted to solve this issue once and for all (in MkIV we has several methods but in LMTX we only have one left). We not only added a native prime element to nuclei but also introduced font parameters similar to those of superscripts. Just keep in mind that in LuaMetaT_EX we don't need to worry about having a few more fields in a math node. There are not that many nodes involved and the amount of extra memory used can be neglected. Of course there is plenty of code added to deal with it so the binary definitely is larger due to it and the code way more complex. In fact we already introduced some missing parameters for the spacing related to prescripts. Furthermore we can tune the prime spacing relative to the scripts.

In the goodie files you can add fixes that relate to primes and most of them involved quite a bit of experimental. We can safely say that we spent quite some prime time on this issue.

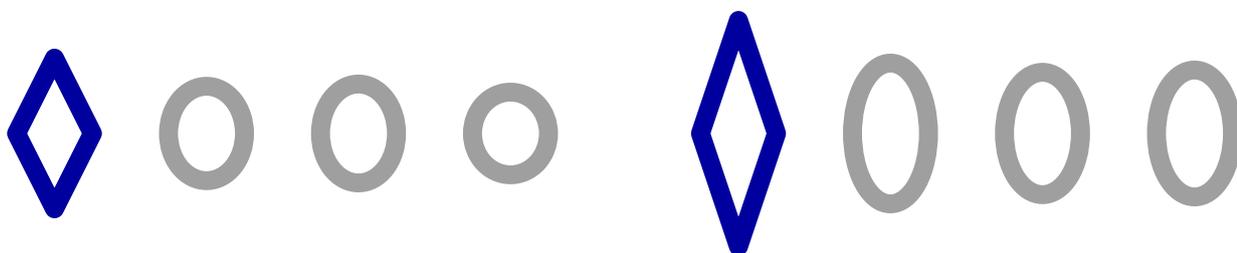
- It would be nice if fonts at least has prime shapes that are consistent because currently script one can look quite different (tilt and shape).
- Because primes are also minutes and seconds we probably have to accept the current situation and deal with it in the goodie files forever.

Two choices

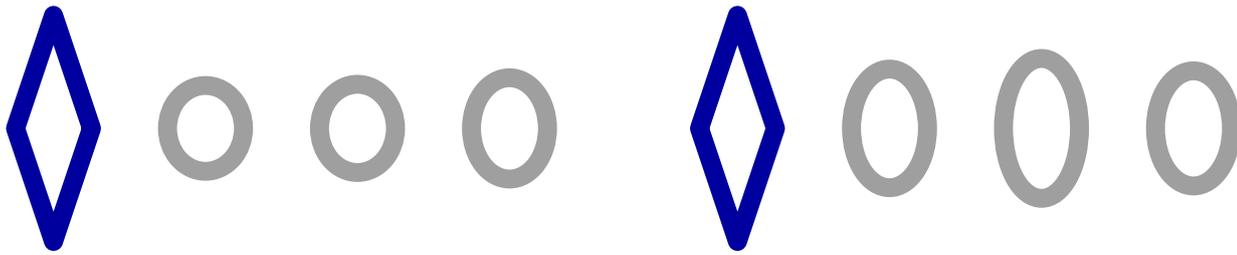
Imagine a situation like this: a sequence of characters with a leading left fence but nothing at the right. The fence is not really a fence but something that can grow and have its own super and subscripts, either after it or on top and below.



There is a visual aspect here, which can be illustrated from a variant:



One can argue that the larger characters in the second sequence rightfully trigger a larger blue variant. But for consistency one might actually go for:



Now imagine that you only have two choices? When do you go for the larger one? This situation occurs with so called large operators like integrals and summations. The original \TeX fonts have two sizes of these characters. The smaller one is used in normal (text) math and the larger one in display math.

But what if, as in OpenType fonts there are more? There is a font parameter `DisplayOperatorMinHeight` that tells what size to use (as a minimum) in display mode. Because the \TeX engines never had to make a decision the value of that variable in the Latin Modern and Gyre fonts is somewhat arbitrary. It is one of the variables that we need to adapt in the goodie files: we roughly bump from 1300 to 1800 to get what we are accustomed to in display mode.

So what are these large operators anyway? In traditional \TeX they are just that: larger variants of smaller ones. In OpenType math however, they are extensibles. That means that when there are more variants and an extensible recipe it can grow on demand. That conflicts with the expectations of two sizes.

In order to support these two conflicting demands LuaMeta \TeX provides so called left operators that either act upon their own, in which case we talk of ‘auto’ mode, or they can be told to have a specific size, or they can adapt, in which case they have the usual bogus right companion that ends the subformula. And, because they are implemented as fences but are not really fences there are some provisions for the scripts: they bind to the left fence and not the right one.

As side note: when we were experimenting with the usage of these (new) mechanisms we again ran into the race condition that is imposed by the algorithm that determines the size of fences: the delimiters target height is either a fraction of the total height of the sublist or the total height diminished by some constant amount. In LuaMeta \TeX we therefore added the already discussed `\UmathDelimiterPercent` and `\UmathDelimiterShortfall` but these only kick in when we have a wrapped integral or such.

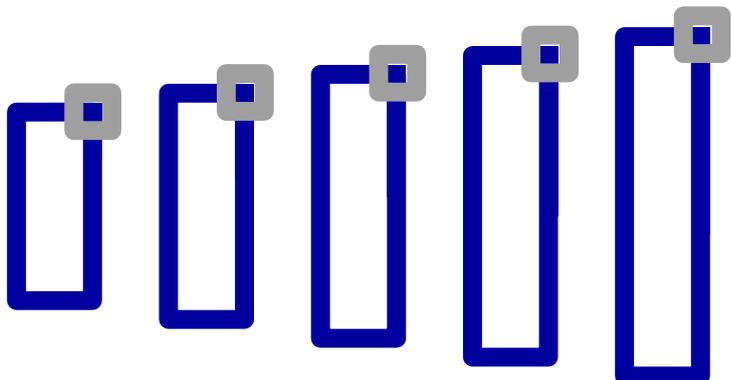
- We can't be too picky but it would be nice if fonts have `DisplayOperatorMinHeight` set to a more reasonable value. We have to tweak most font for this now (and probably forever).

To the point

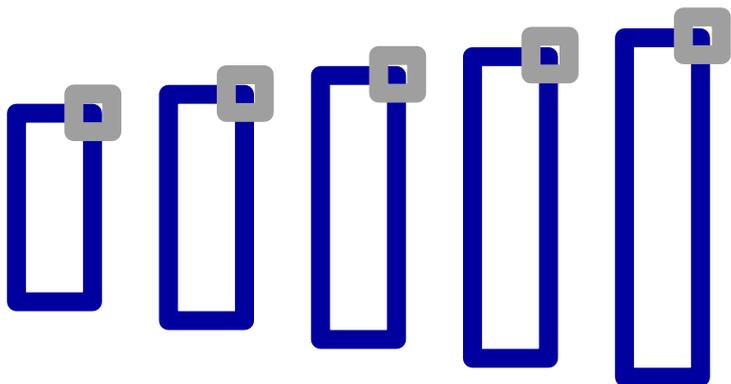
A nice feature of math fonts is that they have so called extensible characters. Fences can grow vertically and accents horizontally. When we run out of discrete steps in size, we end up with a recipe that build large characters from snippets as we discussed before.

The limitations in the size of fonts and the fact that the engine also had to impose some limits in the amount of used fonts and complexity of their usage made for some exceptions to the rule and, no pun intended, it might be why \TeX has a concept of rules. Lines over and under something, arrows, fractions and radicals all use rules. It is interesting to notice that when OpenType fonts showed up the converted \TeX fonts didn't make arrows and bars use the extension mechanism, thereby forcing the old school construction of them. This doesn't hurt Con \TeX t much because we can implement proper characters using the virtual character mechanism and we did things different anyway (for instance we can kick in MetaFun replacements).

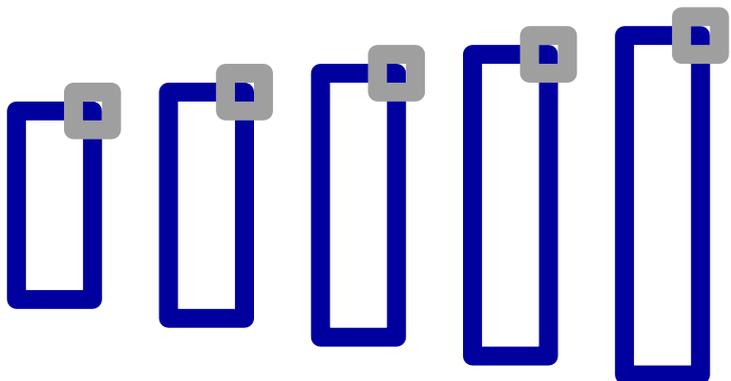
Nevertheless there is this curious in-between: radical. It comes in a set of fixed sized often sloped characters and then switches to an upright one. In all cases there is that horizontal rule attached that covers the nucleus. There is no mix of horizontal and vertical extensibles so there is no fancy end of rule thingie possible (the little hook that we learned to draw at school): it's just a stupid rule.



Instead of a rule, these examples show a simple rectangle attached to another one. The idea is that the center of that small one snaps onto the upper right corner of the large one. In these examples we show how it looks when that is not exact. At small sizes it goes unnoticed but at larger ones it becomes visible. Actually, when you start scaling up math you often see artifacts in shapes, which makes one wonder if high resolution screens are really used for proofing.



In the first example the effect is small because we make sure that the right top corner is square but in the second one we don't do that. Now it really becomes annoying, even if we remove the small errors in positioning.



We have seen different effects. The most common was an inaccurate font dimension that determines what the height of the rule is. That we can fix in the goodie file and we do that. Other artifacts were (in the stepwise sizes) characters that were so sloped that there was not possibility for properly attaching the rule (we noticed

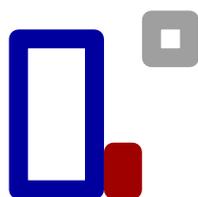
this best when checking the Lucida and Latin Modern fonts): one can cheat and move a little to the left but the thickness of the sloped line was not enough for that, so it could qualify as a design error but a design cannot simply be changed in such case: one has to compromise and assume no scaling (keep in mind that we scale on screen while on print one has to take a magnifying glass and reading math that way is not much fun).

Some of these extensible radicals have rather thin vertical lines but we can assume that their height is never such that this becomes too annoying. One complication in designing these shapes is that when one uses a font creator program it is unlikely to provide a math composer for testing. The same is true for overlapping the end glyphs of e.g. curly braces: the middle piece can get into the way when we have the smallest extensible and have that one kick in too soon due to the lack of stepwise sizes. It's hard to check that without an engine applying them to the extreme cases.

- Benefit from your high resolution screen and zoom in on the results. Take no risk.
- Play safe and create an top right corner that has some overlap and already starts out horizontally.
- If the design complicates rule attachments, move to an extensible sooner because that one can have a horizontal attachment point as part of the design.

Removing slack

In the process of optimizing the spacing we ran into several cases where a bit of spacing was added that eventually made that the resulting formula had small quite visible whitespace at the edges. Take for instance the little bit of kerning between a nuclues and a superscript.



Normally one can predict this situation but when components are glued together that are typeset independently the (in this case red) spacing remains. The engine is capable to remove that kind of slack at the left and right of the formula so that one gets a proper tight bounding box. There is a bit more logic on board now.

The fact that all kind of spacing gets added is one of the reasons for the more granular inter atom spacing: that way we could remove for instance the spacing added to the left and right of fractions (which is buried in the constructed box).

- There is not much we can advice here because it doesn't relate to fonts. However, given that Lucida text fonts have no kerning could make one wonder to what extend math fonts needs all these compensations.
- Some of the inter atom/script kerning is probably there as safeguard for italic shapes where staircase kerns are lacking. The question then is how the related font parameters themselves relate to staircase kerns. We suppose that this puts some stress on the font designer.

Initializing fonts

introduction

When a math font is initialized there are (as with any font) features applies. The only official one is the `ssty` feature that defines the substitution sets for script and scriptscript size. In traditional \TeX one has to set up multiple fonts because one can only have 256 characters and because one might want to map alphabets on the regular ascii slots. In an OpenType font all these alphabets are on the same font so basically one can do with three instances per size: text, script and scriptscript. What actually is done depends on the philosophy of the macro package but we will not discuss that here. So let's assume three instances in the default case. In LuaMeta \TeX we can scale on the fly and therefore we can stick to one instance: as long as we know what the slots of the (optionally provided) smaller sizes are. Supporting this had quite some consequences for the engine as scaling happens all over the place (also think of font parameters) and we're talking two independent dimensions here: horizontal and vertical sizing.

In addition to this size related font feature Con \TeX t provide some additional ones (and always had). Some are set up as regular features and some are driven by the font goodie features which organizes a lot of features under one umbrella. Font goodies always have been part of MkIV and LMTX.

also mention the non tweak related ones

wipeitalics

This tweak wipes the italics from one or more characters, most noticeable upright characters. Because locating the individual characters took too much time, we added the option complete alphabets.

example

wipeanchors

This tweak removes the top anchors from one or more characters, most noticeable upright characters. As with wiping italics, we often wipe complete alphabets.

example

accentdimensions

Configure the position of overbar, underbar, overbrace, underbrace, overparent, underparent, overbracket and underbracket. More? Added 220307.

addrules

I think we did not get overbars for some fonts (kpfonts and erewhon come to mind).

addscripts

Maybe something done for Ton? (`math-act`)

checkspacing

I don't know how it works. Seems there is not really any settings. But you once wrote

```
\liminf$ \quad  
\limsup$ \quad [\sixperemspace] \quad  
$x\sixperemspace x$  
  
\setbox0\hbox{$x\sixperemspace x$} \showbox0
```

in an email (220118).

dimensions

We use this to change anchor points of accents, to raise/lower them, and of course to change bounding boxes and italic correction for many letters/symbols. We use it in bonum to resize the whole lower case fraktur alphabet (and maybe more should be added)

fixprimes

Maybe explained elsewhere? But it could belong here.

fixradicals

I don't know what it does. Could be: In some fonts the radicals are just positioned wrongly, and this moves them up/down to have something uniform to work with. (`math-act`)

kerns

A bit like staircase kerning. The 'topleft' and 'bottomright' come in handy, in particular the last one when it comes to the position of subscripts.

margins

We fake the width of some characters. Useful, for example, not to get too large/too small accents when using `\widehat` (also to get a uniform size over some alphabets or glyphs that often go together).

variants

Some fonts (lucida, xits, stixtwo) have calligraphic (chancery) and script (roundhand) alphabets. Of course not the same as default, and of course not the same `ss0X`.

wipecues

Only added to cambria and dejavu. Added 220327. I think it has to do with characters like 2061, 2062 and 2063. From the mails I think it has why does 2061 render a shape in some fonts so we need an 'wipe char' tweak (basically making it a zero dimensions symbol)

```
\mathspacingmode2
```

```
\showmakeup[mathglue]
```

```
$ \sin          \Uchar"2061 (y) $ \quad  
$ x            \Uchar"2062  y  $ \quad  
$ x            \Uchar"2063  y  $ \par  
$ \sin        \mathghost{\Uchar"2061} (y) $ \quad  
$ x           \mathghost{\Uchar"2062}  y  $ \quad  
$ x           \mathghost{\Uchar"2063}  y  $ \par
```

bigslots

Fences are chosen automatically to match what they surround. However, in traditional engines a fenced sub formula won't break across lines. In ConTEXt we have several st

1, 3, 5, 7 that could fit here, if we find a font where the linking is not present.

parameters

I don't know if all of them are parameters that typically moved from document to font level.

20 Constants

Strings don't really fit into the concept of T_EX. There everything we input and store is tokens and nodes, so when you define a macro like

```
\def\foo{foo}
```

you don't store a string but a tokenlist with three tokens:

control sequence: foo					
827212	11	102	letter	f	U+00066
596486	11	111	letter	o	U+0006F
923274	11	111	letter	o	U+0006F

We have three single byte characters but end up with 32 bytes memory used because we have a linked list with a housekeeping initial token; such a token has a value (operator & operand) as well as a pointer to a next token. This is quite ok because whenever we need that macro the body has to be interpreted and it already being tokenized is what makes T_EX fly.

There are occasions where the expansion of a list that itself can contain references to macros produces a new list in which case copies are being made. Take this:

```
\def\oof{foo}  
\def\foo{foo \oof}
```

control sequence: foo					
931576	11	102	letter	f	U+00066
724862	11	111	letter	o	U+0006F
714540	11	111	letter	o	U+0006F
924069	10	32	spacer		
930368	140	0	call		oof

When `\foo` is expanded, the macro body is pushed onto the input stack and traversed and when `\oof` is seen, that one gets pushed and processed. No copy is needed. Now take this:

```
\def\oof{foo}  
\edef\foo{foo \oof}
```

control sequence: foo					
923947	11	102	letter	f	U+00066
930251	11	111	letter	o	U+0006F
932080	11	111	letter	o	U+0006F
703845	10	32	spacer		
931683	11	102	letter	f	U+00066
823897	11	111	letter	o	U+0006F
930278	11	111	letter	o	U+0006F

Here `\foo` gets the expanded result but again `\oof` got pushed onto the stack. This doesn't involved copying either but there is still the pushing and popping input overhead. So when does copying occur? Here is an example:

```
\def\oof{oof}
\def\ofo{ofo}
\def\foo{\begincsname \oof:\ofo\endcsname}
```

control sequence: foo

826984	136	2	cs name	begincsname
931766	140	0	call	oof
923547	12	58	other char	: U+0003A
932413	140	0	call	ofo
931806	79	0	end cs name	endcsname

When a csname is checked, the engine needs to construct a string in order to access the hash table. Here is what happens:

- everything upto the `\endcsname` is collected
- in the process macros are expanded (with pushing and popping input) and the expanded tokens are appended to the result
- when we're okay that list get converted to a string
- that string is used as lookup into the hash

Normally we're okay but when there is some unexpected unexpandable token (an assignment, node generator, protected macro, etc.) the collection stops and the list so far is recycled. This process is quite efficient, as is everything \TeX , but given that going from token list to string involved some utf8 juggling too there definitely is some overhead.

In $\text{Con}\TeX$ t we use csname checking and usage quite a lot. The first line is the traditional way. It has the disadvantage that it creates an hash entry with alias `\relax` if there is no such name. That is why $\text{-}\TeX$ came up with the test as in the second line. In $\text{Lua}\TeX$ we introduced `\lastnamedcs` so that we don't have to construct the mentioned) token list again which saves time. The fourth line is similar to the first line but doesn't create a new command.

```
\csname \namespace\key\endcsname ...
\ifcsname \namespace\key\endcsname \csname \namespace\key\endcsname ... \fi
\ifcsname \namespace\key\endcsname \lastnamedcs ... \fi
\begincsname \namespace\key\endcsname
```

One of the things all versions of $\text{Con}\TeX$ t have in common (right from the start) is that we use this namespace model consistently. In MkIV we changed the subsystem that deals with this: it's more flexible and uses less memory but it also has way more overhead. But on the average performance is about the same so users didn't notice that.

There is however a trick to speed this up a bit. In the 360 page $\text{LuaMeta}\TeX$ manual we expand macros like `\namespace` and `\key` 4.3 million times (beginning of June 2023). Because Mikael Sundqvist and I are in the middle of some math magic, we also checked his 300 page math book, and that also does it 4.2 million times (the gain was about 0.5 seconds). The upcoming math manual has some 1.2 million. How come that we have so many expansions? First of all we use abstraction when possible and that means that there's

plenty of checking of options and some constructs fall back on parent classes (sometimes more than two times up the parent chain). Also, we often have three macros to expand:

```
\ifcsname\namespace\currentinstance\key\endcsname
```

But these have an important property: their body is basically a string. Nothing in there needs expansion and if it does, it's an indication of rubbish that doesn't contribute for a valid csname anyway. Once we know that we can improve performance:

```
\cdef\oof{oof}
\cdef\ofo{ofo}
\def\foo{\beginsname \oof:\ofo\endcsname}
```

So, `\cdef` (or `\constant\edef` flags the macro as being a constant that doesn't require expansion. For the record, when you define that macro having arguments it just becomes an `\edef`.

control sequence: foo

924792	136	2	cs name		beginsname
932128	143	0	constant call		oof
931756	12	58	other char	:	U+0003A
930795	143	0	constant call		ofo
932126	79	0	end cs name		endcsname

Here we define the two macros as constant ones which in practice means that they are just macros but also indicates that in some scenarios we can directly use their body. Now when in this csname construction we do this instead:

- everything upto the `\endcsname` is collected
- in the process macros are expanded (with pushing and popping input) but when we have a constant we add reference token when there is more than one body token, otherwise the expanded tokens are appended to the result
- when we're okay that list get converted to a string and in that stage we just convert the referenced body of the constant
- that string is used as lookup into the hash

So, instead of immediately injecting an expanded body of a macro that needs no expansion we inject a reference and use that later on for the conversion into characters. On the 4242938 times in LuaMetaTeX (at the time of writing this) this trick gives the following results.

```
\edef\foo{xxxx} \beginsname\foo\endcsname 0.37
\cdef\foo{xxxx} \beginsname\foo\endcsname 0.28
\edef\foo{xxxx} \ifcsname\foo\endcsname\fi 0.53
\cdef\foo{xxxx} \ifcsname\foo\endcsname\fi 0.35
```

And here for an existing command (`\relax`):

```
\edef\foo{relax} \beginsname\foo\endcsname 0.55
\cdef\foo{relax} \beginsname\foo\endcsname 0.36
\edef\foo{relax} \ifcsname\foo\endcsname\fi 0.62
\cdef\foo{relax} \ifcsname\foo\endcsname\fi 0.36
```

When I used that trick in for instance some font switching macros it also had some gain. For instance 200000 times `\it` went from 0.60 down to 0.54 seconds but it is unlikely that in a document one does that many font switches.³⁰

In practice other operations play a role, so here we might also benefit from the data being in the cpu cache but on the manual I gained a decent .2 seconds. One can question if on a 8.5 second run this is worth the trouble. However, in this particular manual we spend 3.5 seconds on font processing, some 1.5 seconds on the backend and have a unique MetaPost graphics on every page. We spend more time in Lua than in T_EX! On 4 seconds T_EX, these .2 seconds is some 2.5 gain, and it might actually be even more percent wise.

In case one wonders why I spend time on this, one reason is that the last decade I was not that impressed by performance gains of a single core and T_EX is a single core process. I also can't afford the latest greatest laptops and definitely don't want to contribute more e-waste. Also, with T_EX and friends running on virtual machines and competing for resources (memory, cpu and disk or network drives) any gain is good gain. Of course it is also fun to improve LuaMetaT_EX and this string-like property has always bothered me.³¹

³⁰ There are a few more places where constants can gain a little but those don't add up much.

³¹ I did some experiment with a native string register but that made no sense because then tokenization in other places takes a toll. With the mentioned constants we don't pay that price.

21 Active characters

Each character in \TeX has a so called category code. Most are of category ‘letter’ or ‘other character’ but some have a special meaning, like ‘superscript’ or ‘subscript’ or ‘math shift’. Of course the backslash is special too and it has the ‘escape’ category.

A single character can also be a command in which case it has category ‘active’. In $\text{Con}\TeX$ the `|` is an example of that. It grabs an argument delimited by yet another such (active) bar and handles that argument as compound character.

From the perspective of $\text{Con}\TeX$ we have a couple of challenges with respect to active characters.

- We want to limit the number of special symbols so we only really have to deal with the active bar and tilde. Both have a history starting with MkII.
- There are cases where we don't want them to be not active, most noticeably in math and verbatim. This means that we either have to make a sure that they are not active bit in nested exceptions, for instance when we flush a page halfway verbatim, made active again.
- In text we always had catcode regimes to deal with this (which is actually why in $\text{Lua}\TeX$ efficient catcode tables were one of the first native features to implement. This involves some namespace management.
- In math we have to fall back on a different meaning which adds another (meaning) axis alongside catcode regimes: in math we use the same catcode regime as in text so we have a mode dependent meaning on top of the catcode regime specific one.
- In math we have this special active class/character definition value `"8000` that makes characters active in math only. We use(d) that for permitting regular hat and underscore characters in text mode but let them act as superscript and subscript triggers in math mode.
- Active characters travel in a special way trough the system: they are actually stored as macro calls in token lists en macro bodies. This normally goes unnoticed (and is not that different from other catcodes being frozen in macros).

So far we could always comfortably implement whatever we wanted but sometimes the code was not that pretty. Because part of the $\text{LuaMeta}\TeX$ project is to make code cleaner, I started wondering if we could come up with a better mechanism for dealing with active characters especially in math. Among the other reasons were: less tracing clutter, a bit more natural approach, and less intercepts for special cases. Of course we have to be compatible. Some first experiments were promising but as usual it took a while to identify all the cases we have to deal with. At moments I wondered if I should go forward but as I stepwise adapted the $\text{Con}\TeX$ code to the experiment there was no way back. I did however reject experiments that out active characters in the catcode table namespaces.

In $\text{Lua}\TeX$ (and its predecessors) internally active characters are stored as a reference to a control sequence, although a `\show` or trace will report the character as ‘name’. For example:

```
\catcode `!=\activecatcode
\def !{whatever} % we also have \letcharcode
\def\foo{x!x}
```

is stored as (cs, cmd, chr):

control sequence: foo

931943	11	120	letter	x	U+00078	
930787	140	0	call			whatever
827036	11	120	letter	x	U+00078	

However, when we want some more hybrid approach, a text versus math mix, we need to postpone resolving into a control sequence. Examples are macro bodies and token registers. When we flag a character (with `amcode`) as being of a different catcode than active in math mode, we get the following:

```
\amcode`! \othercatcode
\catcode `!=\activecatcode
\def !{whatever}
\def\foo{x!x}
```

control sequence: foo

924150	11	120	letter		x	U+00078
781170	13	33	active char			
931785	11	120	letter		x	U+00078

The difference is that here we get the active character in the body of the macro. Interesting is that this is not something that parser is prepared for so the main loop has now to catch active characters. This is no big deal but also not something to neglect. The same is true for serialization of tokens.

Other situations when we need to be clever is for instance when we try to enter math mode. In math mode we want the (in text) active character as math character and a convenient test is checking the mode. However, when we see `$` we are not yet in math mode and as `TEX` looks for a potential next `$` we grab a active character it should not resolve in a reference to an command. The reason for that is that when `TEX` pushes back the token (because it doesn't see a `$`) we need it to be an active character and not a control sequence. If it were a control sequence we would see it as such in math mode which is not what we intended. It is one of these cases where `TEX` is not roundtrip. Similar cases occur when `TEX` looks ahead for (what makes a) number and doesn't see one which then results in a push back. Actually, there are many look ahead and push back moments in the source.

```
text: \def\foo{x!!|x}
```

```
\meaningasis\foo
```

```
\luatokenable\foo
```

```
$x\foo x$ \foo
```

text:

```
\def \foo {x!!|x}
```

control sequence: foo

827251	11	120	letter		x	U+00078
924528	13	124	active char			
781108	12	33	other char	!	U+00021	

923660	13	124	active char		
932660	11	120	letter	x	U+00078

↑ `x!x`

`math: $\gdef\oof{x||x}$`

`\meaningasis\oof`

`\luatokenable\oof`

`$x\oof x$ \oof`

math:

`\def \oof {x||x}`

control sequence: oof

932494	11	120	letter	x	U+00078
932421	13	124	active char		
930802	12	33	other char	!	U+00021
931001	13	124	active char		
923470	11	120	letter	x	U+00078

↑ `x!x`

`toks: \scrachtoks{x||x}`

`\detokenize\expandafter{\the\scrachtoks}`

`\luatokenable\scrachtoks`

`$x\the\scrachtoks x$ \the\scrachtoks`

toks:

`x||x`

token register: scrachtoks

932138	11	120	letter	x	U+00078
924731	13	124	active char		
931841	12	33	other char	!	U+00021
932436	13	124	active char		
932502	11	120	letter	x	U+00078

↑ `x!x`

A good test case for ConT_EXt is:

`\def\foo{x||x||x}`

`x||x||x + \foo`

```
$x|!|x||x + \foo$
```

Here we expect bars in math mode but the compound mechanism applied in text mode:

```
x!x-x + x!x-x
```

```
|_acode |_bina|_baed |_acode
```

So the bottom line is this:

- Active characters should behave as expected, which means that they get replaced by references to commands.
- When the `\amcode` is set, this signals the engine to delay that replacement and retain the active character.
- When the moment is there the engine either expands it as command (text mode) or injects the alternative meaning based on the catcode. There we support letters, other characters, super- and subscripts and alignment codes. The rest we simply ignore (for now).

Of course you can abuse this mechanism and also retain the character's active property in text mode by simply setting the `\amcode`. We'll see how that works out. Actually this mechanism was provided in the first place to get around the "8000 limitations! So here is another cheat:

```
\catcode `^ \othercatcode      % so a ^ is just that  
\amcode `^ \superscriptcatcode % but a ^ in math signals a superscript
```

So, the `a` in `\amcode` stands for both 'active' and 'alternative'. As mentioned, because we distinguish between math and text mode we no longer need to adapt the meaning of active commands: think of using `\mathtext` in a formula where we leave math mode and then need to use the text meaning of the bar, just as outside the formula.

In the end, because we only have a few active characters and no user ever demanded name spaces that mechanism was declared obsolete. There is no need to keep code around that is not really used any more.

Internally an active character is stored in the hash that also stores regular control sequences. The character becomes a utf string prefixed by the utf value of `0xFFFF` which doesn't exist in Unicode. The `\csactive` primitive is a variant on `\csstring` that returns this hash. Its companion `\expandactive` (a variant on `\expand`) can be used to inject the related control sequence. If `\csactive` is not followed by an active character it expands to just the prefix, as does `\Uchar"FFFF` but a bit of abstraction makes sense.

22 Accuracy

One of the virtues of \TeX is that it can produce the same output over a long period. The original engine only uses integers and although dimensions have fractions but these are just a way to present them to the user because internally they are scaled points.

<code>\dimexpr .4999pt : 2 \relax</code>	0.24994pt
<code>\dimexpr .4999pt / 2 \relax</code>	0.24995pt
<code>\scratchdimen .4999pt \divide\scratchdimen 2 \the\scratchdimen</code>	0.24994pt
<code>\scratchdimen .4999pt \edivide\scratchdimen 2 \the\scratchdimen</code>	0.24995pt
<code>\scratchdimen 4999pt \divide\scratchdimen 2 \the\scratchdimen</code>	2499.5pt
<code>\scratchdimen 4999pt \edivide\scratchdimen 2 \the\scratchdimen</code>	2499.5pt
<code>\scratchdimen 4999pt \rdivide\scratchdimen 2 \the\scratchdimen</code>	2500.0pt
<code>\numexpr 1001 : 2 \relax</code>	500
<code>\numexpr 1001 / 2 \relax</code>	501
<code>\scratchcounter 1001 \divide\scratchcounter 2 \the\scratchcounter</code>	500
<code>\scratchcounter 1001 \edivide\scratchcounter 2 \the\scratchcounter</code>	501

The above table shows what happens when we divide an odd integer or for that matter odd fraction. Note the incompatibility between `\numexpr` and `\dimexpr` on the one hand and `\divide` on the other. This is why in LuaMeta \TeX we have the `:` variant that does the same integer divide (no rounding) as `\divide` does, and why we have `\edivide` that divides like an expression using the `/`. The `\rdivide` only makes sense for dimensions and rounds the result.

As soon as one starts calculating or comparing accumulated values one can run into the values being a few scaled points off. This means that when one tests against a criterium it might be that some range comparison is better. The most likely place for that to happen is in the output routine and when special constructs like floats, tables and images come into play. Just like not every number can be represented in a float (double), we saw that dividing an odd integer can give some unexpected rounding as part of the integer is considered a fraction. So, in practice, even when the calculations are the same, there is a certain unpredictable outcome from the user perspective: “Why does it fit here and not there?” Well, we can be a few scaled points off due to some not entirely round-trip calculation.

When \TeX showed up it came with fonts and in those times once a font was released it was unlikely to change. But today fonts do change. And changes means that a document can render differently after an update. Of course this is an argument for keeping a font in the \TeX tree but even then updating is kind of normal. Take math: the fact that fonts often have issues makes that we need to tweak them and some tweaks only get added when we run into some issue. If that issue has been there for a while we are incompatible.

Hyphenation patterns are another source of breaking compatibility but normally they change little. And here one can also assume that the user wants words to be hyphenated properly. Even with such fundamental changes as a syllable being able to move to the next line, it is often unlikely that the paragraphs get less or more lines. I bet that users are more worried about the impact on vertical rendering that has consequences for page breaks than for lines coming out differently (hopefully better).

So, what are other potential areas in addition to slight differences due to division, fonts and patterns? We now enter the world of LuaMeta \TeX and Con \TeX t. As soon as one starts to use Lua code, doubles show up. It means that we can do calculation with little loss because a double can safely hold the maximum dimension

(in scaled point). However, mixing 64 bit doubles at the Lua end with 32 bit integers in the engine can have side effects. As soon as set some property at the T_EX end using Lua rounding takes place. Of course we can do all calculations like T_EX does, but that would have too much of an impact on performance.

So, going back and forth between T_EX and Lua can introduce some inaccuracies creeping in but as long as it is consistent, there is no real issue. It mostly involves fonts and especially the dimensions of characters: the width, height and depth but when one uses the xheight as relative measure there is also some influence on for instance interline spacing, offsets and such.

So how can fonts make a difference? In ConT_EXt there are two ways to use fonts: normal mode and compact mode. In normal mode every size is an instance, where the dimension properties of characters are scaled. In compact mode we use one size and delegate scaling to the engine which means that we end up with the (usual) 1000 being scale 1 kind of calculations. In the end a font with design size of 10bp (most fonts) scaled to 12pt normal is not behaving the same as a 10pt setup where a 12pt size is scaled on demand. First there is the scaling from 10bp loaded font to the 10pt used font that gets passed to T_EX. Here we have to deal with history: defining a font in pt points is quite normal. Then applying a 1200 scale (later divided by 1000) in the engine again involves some rounding to integers because that is what is used internally. I will come back to this later.

The main conclusion to draw is that normal mode and compact mode come close but give different results. We can come closer when we a more accurate normal mode. In order to limit the number of font instances we normally limit the number of digits (also in compact mode but there accuracy comes a little cost). There is a pitfall here: While T_EX can happily work with any resolution, the backend has to make sure that embedded fonts get scaled right and that (in the case of pdf) we compensate for drift in the page stream, because there character widths determine the advance and these are in (often rounded) bp (big postscript points). Especially when we enable font expansion drift prevention comes with a price as there we are dealing with real small difference in dimensions.

As an experiment I played with clipping measures in the engine which boils down to rounding the last digit but that didn't work out well. For simple text we can get normal and compact mode identical but kick in some math (many parameters involved), font expansion and/or protrusion, additional inter-character kerning and so on, and one never get the same output. Keep in mind that we are not talking visual differences here, although there can be cases. More think of due to a slightly different vertical spacing triggering a different page break, for instance when footnotes are involved. In ConT_EXt the line height (and therefore derived parameters) is defined in terms of the xheight so even a few scaled points off makes a difference.

At the user level, currently compact mode is enabled with:

```
\enableexperiments[fonts.compact]
```

It works quite okay already for years (writing end 2023) in most scenarios but there might be cases where existing code still needs to be adapted, which is no big deal. The additional overhead is compensated by loading less font instances and a smaller output file. In some cases documents actually process faster and it definitely pays of for large fonts (cjk) and demanding mix size feature processing.

A more accurate normal mode is set by:

```
\enableexperiments[fonts.accurate]
```

but it doesn't bring much. It was introduced in order for Mikael Sundqvist and me to compare and check math tweaks, especially those that depend on precise combinations of glyphs. We temporary had some additional control in the engine but after experiments and comparing variants the decision was made to remove that feature.

We ran experiments with large documents where different versions were overlaid and depending on scenarios indeed there can be differences, but when there are chapters starting on new pages and when vertical spacing has stretch, there are not that many differences. When you compare the so called `tuc` files you might notice small difference is position tracking but these values are seldom used in a way that influences the rendering of text, line and page breaks.

To come back to the bp vs pt issue. Among the options considered are moving the character and font properties from integers to doubles, but that would impact the memory footprint quite a bit. Another idea is that compact mode goes 10bp instead of 10pt but that would not help. One bp is 657817 scaled points and one pt is 655360 sp. The ratio between them is 1.0037490844727, so a \TeX scale 1200 effectively becomes 1204.499, and assuming rounding to an integer we then get 1204. So in the end we get a less fortunate number instead of 1200 and it's not even accurate. Therefore this option was also rejected. For the record: an intermediate approach would have been to cheat: use an internal multiplier (the shown ratio) and although it is not hard to support, it also means that at the Lua end we always need to take this into account, so again a no-go.

In the end the only outcome of this bit of 'research' has been that we can have accurate normal font handling (which is not that useful) and have two additional divide related primitives that might be useful and add some consistency (and these might actually get used).

23 Characters in math

This chapter goes into some more details about math characters but after some remarks goes on about about discretionaries. Traditionally \TeX users enter ascii characters mixed with commands and expect to get the right visual representation. Because there are plenty math characters outside the ascii range this means that most are accessed by a command. However, Unicode changes that: when an editor can show the character there is no reason not to use that feature. In that case we end up with utf characters in the input. In this perspective is it important to realize that there is a distinction between such a direct utf character and a command, especially when it is defined as follows:

```
\Umathchardef\mathcharacterf 0 \mathordinarycode `f

\startformula
  f = \mathcharacterf = \Uchar"1D453
\stopformula
```

This gives the expected:

f

The ascii `f` will eventually become character `U+1D453` but let's not worry here about how that is done; what is more important is that this character has some extra properties. Just like the definition of the command we use a primitive `\Umathcode` that registers that we have an ordinary and that the origin is family zero, we also need to make sure that the `U+1D453` has those properties. The way it works is that the engine injects a math noad with a math character nucleus and when it does that it needs to resolve the family and the class. Depending on the style the family will resolve in a text, script or scriptscript font. The class determines the spacing and some specific engine behavior.

In $\text{Con}\TeX$ t and therefore in $\text{LuaMeta}\TeX$ we go as step further. There we also have dictionary fields, which makes it possible to adapt properties like the class as we like after the user has entered them. This (experimental) features relates to the fact that often Unicode math, \TeX character names, and usage doesn't really reveal what the character is about and if it is needs to have class binary, relation or something else. If the command does carry some meaning it gets lost when we end up with these injected math characters. In $\text{LuaMeta}\TeX$ we do carry more around. Because this is experimental and evolving we stick to mentioning that there is for instance a primitive `\Umathdictdef` that does what `\Umathchardef` does but expects three additional numbers: properties, group and index.

In $\text{LuaMeta}\TeX$ we try to be as detailed as possible when we resolve and store references to characters in the math nodes (noads), even if the engine itself doesn't always need that information, for instance: for handling a single character superscript we don't need to know its class.

This detour was needed in order to understand the following: discretionaries in math mode. In $\text{LuaMeta}\TeX$ we are already more tolerant with respect to what can end up in a discretionary and math discretionaries have been supported for a while now. In the next examples, class 2 is used: binary.

```
test $ \dorecurse{50}{\a \discretionary class 2 {\$+\$}{\$+\$}{\$+\$} } b$ test
```

```
test
test
```

But this is not nice: we need to enter math mode in the three snippets and likely also need to make sure that we do that in the right style. So, that was why we can now also do this:

```
test $ \dorecurse{50}{a \mathdiscretionary class 2 {<}>{=} } b$ test
```

```
test
test
```

We can wrap this in a command:

```
\def\weirdrelation{\mathdiscretionary class 2 {<}>{=} }
```

but this is not what we want when we are talking + and - which are candidates for repetition. And these are entered as utf character so there is indication of them being treated special. This is why LuaMetaTeX has a new vector `\hmcode` where one can trigger specific characters to become discretionaries.

```
\hmcode"002B=1 % +
\hmcode"2212=1 % -
```

```
test $ \dorecurse{50}{a + b - } c$ test
```

```
test
test
```

Setting bit one of the code will enable this feature. But as usual with TeX and math there is a pitfall. Take this (unusual) example:

```
\hmcode"1D453=1 % we trigger promotion to discretionary
```

```
test $\dorecurse{50}{a \Umathchar 2 0 "1D453 b} b$ test
```

```
test test
```

We see the **f** being repeated but also notice that the italic correction disappears because that is what happens in the line break. But in math this correction is actually part of the width (we've written plenty about that over the years). However, when we set bit two of the code, the correction is moved into the discretionary:

```
\hmcode"1D453=3 % we carry the italic correction along
```

```
test $\dorecurse{50}{a \Umathchar 2 0 "1D453 b} b$ test
```

```
test test
```

So, where characters need to retain their family and class, we also need to make sure that we retain the fact that a character is to be automatically repeated at a line break. The reason why this ended up in the engine while it could be delegated to a callback is that we do need to process discretionaries in math anyway and

also want to avoid it when we're not at the outer level. And because we already carry around all kind of options with noads and glyphs it was not that hard to support this.

It is a bit of a side track but discretionaries in LuaMetaTeX are a bit more permissive anyway. Take this:

```
\dorecurse{20}{%
  xxxxxx
  \discretionary {>>} {<<} {==}
  xxxxxxxxxxxx
}

xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx ==
xxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx
xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx ==
xxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx
xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx
```

Here we depend on the tolerance and stretch settings in order to not overflow the text boundaries. But how about the next:

```
\dorecurse{20}{%
  xxxxxx
  \discretionary
    {>\hskip0pt plus 5pt>}
    {<\hskip0pt plus 5pt<}
    {=\hskip0pt plus 5pt=}
  xxxxxxxxxxxx
}

xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx ==
xxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx
xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx ==
xxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx
xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx
```

This time we have some glue in the snippets. But we can even do the next trickery, where we can stretch the boxed content after the line break routine has done it work. It is this mechanism that we use deep down in the math engine too.

```
\dorecurse{20}{%
  xxxxxx
  \discretionary
    {\uleaders \hbox to 2em{\>\hss>}\hskip0pt plus 10pt minus 5pt}
    {\uleaders \hbox to 2em{\<\hss<}\hskip0pt plus 10pt minus 5pt}
    % {\uleaders \hbox to 2em{=\hss=}\hskip0pt plus 10pt minus 5pt}
    {==}
  % xxxxxxxx
  xxxxxxxxxxxx
}

xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx ==
xxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx xxxxxx == xxxxxxxxxxxx
```

XXXXXX == XXXXXXXXXXX XXXXX == XXXXXXXXXXX XXXXX == XXXXXXXXXXX XXXXX == XXXXXXXXXXX XXXXX ==
XXXXXXXXXX XXXXX == XXXXXXXXXXX XXXXX == XXXXXXXXXXX XXXXX == XXXXXXXXXXX XXXXX == XXXXXXXXXXX
XXXXXX == XXXXXXXXXXX XXXXX == XXXXXXXXXXX

So, in some way, extending the math engine lets features trickle back into the text engine and vice versa. It is all about seeing (weird) opportunities because it is often after playing with this that one sees more potential.

24 Standardizing math fonts

24.1 Introduction

ConT_EXt has always had a good support for the typesetting of mathematics. ConT_EXt MkII uses the pdfT_EX engine and hence traditional (Type1) fonts. Several math fonts are available, specifically designed to work seamlessly with T_EX. ConT_EXt MkIV, the successor version, utilizes the LuaT_EX engine, providing support not only for traditional fonts but also for OpenType Unicode math fonts. Unlike the X_YT_EX engine, which interpreted these new fonts in a manner similar to traditional T_EX fonts, LuaT_EX adheres more closely to the (unfortunately somewhat vague) OpenType specification.³² When new fonts appeared, some were more like the traditional fonts, others more like OpenType Unicode math fonts. This leads to difficulties in achieving consistent results across different fonts and might be one reason that the Unicode engines are not yet used as much as they probably should.

In autumn 2021 we started to discuss how to improve the typesetting of OpenType Unicode mathematics, and it was natural to go on and do this for the LuaMetaT_EX engine, and hence for ConT_EXt LMTX. Since then, we have been engaging in daily discussions covering finer details such as glyphs, kerning, accent placement, inter-atom spacing (what we refer to as math microtypography), as well as broader aspects like formula alignment and formula line breaking (math macrotypography). This article will primarily focus on the finer details. Specifically, we will explore the various choices we have made throughout the process. The OpenType Unicode math specification is incomplete; some aspects are missing, while others remain ambiguous. This issue is exacerbated by the varying behaviors of fonts.

We make runtime changes to fonts, and add a few additional font parameters that we missed. As a result, we deviate from the standard set by Microsoft (or rather, we choose to interpret it in our own way) and exercise the freedom to make runtime changes to font parameters. Regarding this aspect, we firmly believe that our results often align more closely with the original intentions of the font designers. Indeed, the existence of “oddities” in these fonts may be attributed to the lack of an engine, during their creation, that supported all the various features, making testing difficult, if not essentially impossible. Within ConT_EXt LMTX, we have the necessary support, and we can activate various helpers that allow us to closely examine formulas. Without them our work would not have been possible.

Ultimately, we hope and believe that we have made straightforward yet effective choices, rendering the existing OpenType Unicode math fonts usable. We hope that this article might be inspiring and useful for others who aim to achieve well-designed, modern math typesetting.

24.2 Traditional vs. OPENTYPE math fonts

There is a fundamental difference between traditional T_EX math fonts and OpenType Unicode fonts. In the traditional approach, a math setup consists of multiple independent fonts. There is no direct relationship between a math italic and an on top of it. The engine handles the positioning almost independently of the shapes involved. There can be a shift to the right of triggered by kerning with a so-called skew character but that is it.

A somewhat loose coupling between fonts is present when we go from a base character to a larger variant that itself can point to a larger one and eventually end up at an extensible recipe. But the base character and

³² See <https://learn.microsoft.com/en-us/typography/opentype/spec/math>

that sequence are normally from different fonts. The assumption is that they are designed as a combination. In an OpenType font, variants and extensibles more directly relate to a base character.

Then there is the italic correction which adds kerns between a character and what follows depending on the situation. It is not, in fact, a true italic correction, but more a hack where an untrue width is compensated for. A traditional \TeX engine defaults to adding these corrections and selectively removes or compensates for them. In traditional \TeX this fake width helps placing the subscript properly while the italic correction is added to the advance width when attaching subscripts and/or moving to the next atom.

In an OpenType font we see these phenomena translated into features. Instead of many math fonts we have one font. This means that one can have relations between glyphs, although in practice little of that happens. One example is that a specific character can have script and scriptscript sizes with a somewhat different design. Another is that there can be alternate shapes for the same character, and yet another is substitution of (for instance) dotted characters by dotless ones. However, from the perspective of features a math font is rather simple and undemanding.

Another property is that in an OpenType math font the real widths are used in combination with optional italic correction when a sequence of characters is considered text, with the exception of large operators where italic correction is used for positioning limits on top and below. Instead of abusing italic corrections this way, a system of staircase kerns in each corner of a shape is possible.

Then there are top (but not bottom) anchor positions that, like marks in text fonts, can be used to position accents on top of base characters or boxes. And while we talk of accents: they can come with so-called flat substitutions for situations where we want less height.

All this is driven by a bunch of font parameters that (supposedly) relate to the design of the font. Some of them concern rules that are being used in constructing, for instance, fractions and radicals but maybe also for making new glyphs like extensibles, which is essentially a traditional \TeX thing.

So, when we now look back at the traditional approach we can say that there are differences in the way a font is set up: widths and italic corrections, staircase kerns, rules as elements for constructing glyphs, anchoring of accents, flattening of accents, replacement of dotted characters, selection of smaller sizes, and font parameters. These differences have been reflected in the way engines (seem to) deal with OpenType math: one can start with a traditional engine and map OpenType onto that; one can implement an OpenType engine and, if needed, map traditional fonts onto the way that works; and of course there can be some mix of these.

In practice, when we look at existing fonts, there is only one reference and that is Cambria. When mapped onto a traditional engine, much can be made to work, but not all. Then there are fonts that originate in the \TeX community and these do not always work well with an OpenType engine. Other fonts are a mix and work more or less. The more one looks into details, the clearer it becomes that no font is perfect and that it is hard to make an engine work well with them. In LuaMeta \TeX we can explicitly control many of the choices the math engine makes, and there are more such choices than with traditional \TeX machinery. And although we can adapt fonts at runtime to suit the possibilities, it is not pretty.

This is why we gradually decided on a somewhat different approach: we use the advantage of having a single font, normalize fonts to what we can reliably support, and if needed, add to fonts and control the math engine, especially the various subsystems, with directives that tell it what we want to be done. Let us discuss a few things that we do when we load a math font.

24.3 Getting rid of italic corrections

OpenType math has italic corrections for using characters in text and large operators (for limits), staircase kerns for combining scripts, and top anchor for placement of accents. In LuaMetaTeX we have access to more features.

Let's remind ourselves. In a bit more detail, OpenType has:

- `n \typ{italic correction}` is injected between characters in running text, but: a sequence of atoms is `\em not` text, they are individually spaced. `\stopitem \startitem n italic correction` value in large operators that reflects where limits are attached in display mode; in effect, using the italic correction as an anchor.
- **Top anchors** are used to position accents over characters, but not so much over atoms that are composed from not only characters.
- **Flat accents** as substitution feature for situations where the height would become excessive.
- **Script and scriptscript** as substitution feature for a selection of characters that are sensitive for scaling down.

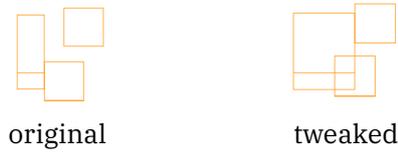
This somewhat limited view on math character positioning has been extended in LuaMetaTeX, and we remap the above onto what we consider a bit more reliable, especially because we can tweak these better. We have:

- **Corner kerns** that make it possible to adjust the horizontal location of sub- and superscripts and pre-scripts.
- Although **flat accents** are an existing feature, we extended them by providing additional scaling when they are not specified.
- In addition to script sizes we also have **mirror** as a feature so that we can provide right to left math typesetting. (This also relates to dropping in characters from other fonts, like Arabic.)
- In addition to the **top anchors** we also have **bottom anchors** in order to properly place bottom accents. These are often missing, so we need to construct them from available snippets.
- An additional **extensible italic correction** makes it possible to better anchor scripts to sloped large operators. This is combined with keeping track of **corner kerns** that can be specified per character.
- Characters can have **margins** which makes it possible to more precisely position accents that would normally overflow the base character and clash with scripts. These go in all four directions.
- In order to be able to place the degree in a radical more precisely (read: not run into the shape when there is more than just a single degree atom) we have **radical offsets**.

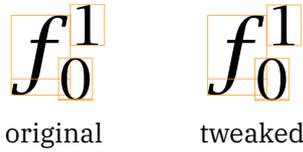
There are plenty more tuning options but some are too obscure to mention here. All high level constructors, like fences, radicals, accents, operators, fractions, etc. can be tuned via optional keyword and key/values at the macro end.

We eliminate the italic correction in math fonts, instead adding it to the width, and using a negative bottom right kern. If possible we also set a top and bottom accent anchor. This happens when we load the font. We also translate the italic correction on large operators into anchors. As a result, the engine can now completely ignore italic corrections in favor of proper widths, kerns and anchors. Let us look at a few examples.

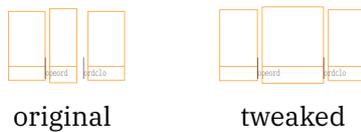
The italic f is used a lot in mathematics and it is also one of the most problematic characters. In TeX Gyre Bonum Math the italic f has a narrow bounding box; the character sticks out on both the left and right. To the right, this is compensated by a large amount of italic correction. This means that when one adds sub- and superscripts, it works well. We add italic correction to the width, and introducing a negative corner kern at the bottom right corner, and thus the placement of sub- and superscripts is not altered. Look carefully at the bounding boxes below.



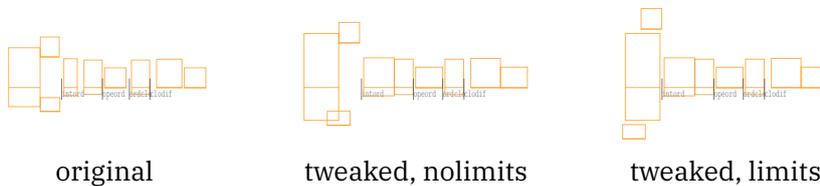
Compare with Lucida Bright Math, which comes with staircase kerns instead of italic correction. We convert these kerns into corner kerns.



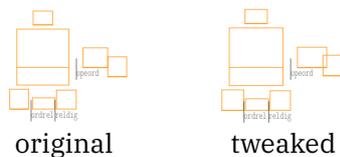
For characters that stick out to the left, we also increase the width and shift the glyph to ensure that it does not stick out on the left side. This prevents glyphs from clashing into each other.



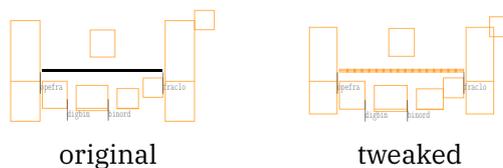
As mentioned, for the integral, one of the most common big operators, the limits are also placed with help of the italic correction. When the limits go below and on top, proper bottom and top anchor points are introduced, calculated from the italic correction. (The difference in size of the integral signs is a side effect of the font parameter `DisplayOperatorMinHeight` being tweaked, as we'll discuss more later. OpenType fonts can come with more than two sizes.)



Compare these integrals with the summation, that usually does not have any italic correction bound to it. This means that the new anchor points end up in the middle of the summation symbol.



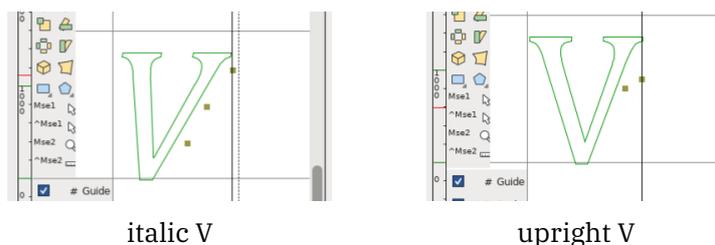
We also introduce some corner kerns in cases where there were neither italic corrections nor staircase kerns. This is mainly done for delimiters, like parentheses. We can have a different amount of kerning for the various sizes. Often the original glyph does not benefit from any kerning, while the variants and extendibles do.



Note also the different sizes of the parentheses in the example above. Both examples are set with `\left(` and `\right)`, but the font parameters are chosen differently in the tweaked version. Font designers should have used the opportunity to have more granularity in sizes. Latin Modern Math has four, many others have steps in between, but there is a lack of consistency.

24.4 Converting staircase kerns

We simplify the staircase kerns, which are often somewhat sloppy and seldom complete (see figure below), into more reliable corner kerns. It's good enough and looks better on the whole. We also avoid bugs that way.



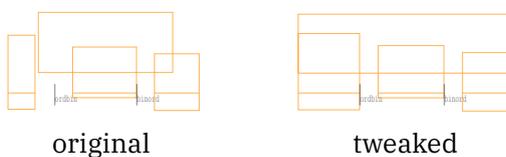
24.5 Tweaking accents

We ignore the zero dimensions of accents, simply assuming that one cannot know if the shape is centered or sticks out in a curious way, and therefore use proper widths with top and bottom anchors derived from the bounding box. We compensate for negative `llx` values being abused for positioning. We check for overflows in the engine. In case of multiple accents, we place the first one anchored over the character, and center the others on top of it.



We mentioned in an earlier TugBoat article that sometimes anchor points are just wrong. We have a tweak that resets them (to the middle) that we use for several fonts and alphabets.

Some accents, like the hat, can benefit from being scaled. The fonts typically provide the base size and a few variants.



The only fonts we have seen that support flattened accents are Stix Two Math and Cambria Math.



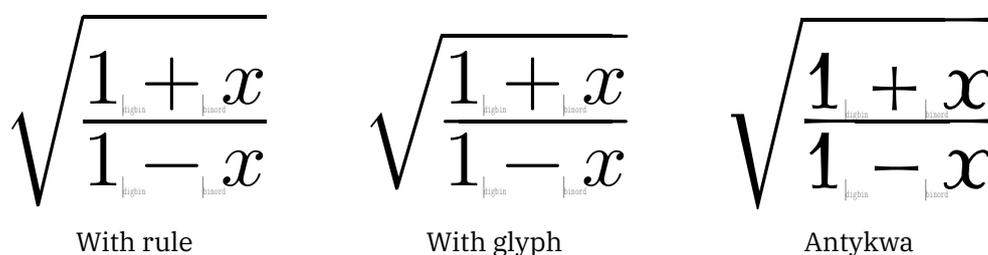
If you look carefully, you notice that the hats over the capital letters are not as tall as the one over the lower-case letter. There is a font parameter `FlattenedAccentBaseHeight` that is supposed to specify when this effect is supposed to kick in. Even though other fonts do not use this feature, the parameter is set, sometimes to strange values (if they were to have the property). For example, in Garamond Math, the value is 420.

We introduced a tweak that can fake the flattened accents, and therefore we need to alter the value of the font parameter to more reasonable values. We communicated to Daniel Flipo, who maintains several math fonts, that the parameter was not correctly set in Erewhon math. In fact, it was set such that the flattened accents were used for some capital letters (C in the example below) but not for others (A below). He quickly fixed that. The green rules in the picture have the height of `FlattenedAccentBaseHeight`; it did not need to be decreased by much.



24.6 Getting rid of rules

We get rid of rules as pseudo-glyphs in extensibles and bars. This also gives nicer visual integration because flat rules do not always fit in with the rest of the font. We also added support for this in the few (Polish) Type 1 math fonts that we still want to support, like Antykwa Toruńska.



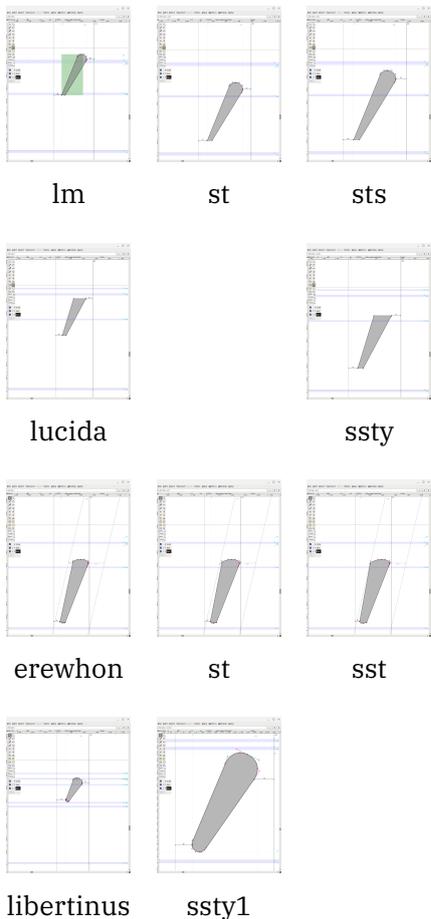
Here is an enlarged example of an Antykwa rule. Latin Modern has rounded corners, here we see a rather distinctive ending.



24.7 Tweaking primes

We make it no secret that we consider primes in math fonts a mess. For some reason no one could convince the Unicode people that a ‘prime’ is not a ‘minute’ (that is, U+2032 PRIME is also supposed to be used as the symbol for minutes); in case you'd like to argue that “they often look the same”, that is also true for the Latin and Greek capital ‘A’. This lost opportunity means that, as with traditional T_EX fonts, we need to fight a bit with placement. The base character can or cannot be already anchored at some superscript-like position, so that makes it basically unusable. An alternative assumption might be that one should just use the script size variant as a superscript, but as we will see below, that assumes that they sit on the baseline so that we can move it up to the right spot. Add to that the fact that traditional T_EX has no concept of a prime, and we need some kind of juggling with successive scripts. This is what macro packages end up doing.

But this is not what we want. In ConT_EXt MkIV we already have special mechanisms for dealing with primes, which include mapping successive primes onto the multiple characters in Unicode, where we actually have individual triple and quadruple primes and three reverse (real) primes as well. However, primes are now a native feature, like super- and subscripts, as well as prescripts and indices. (All examples here are uniformly scaled.)



Because primes are now a native feature, we also have new font parameters `PrimeShiftUp` and `PrimeShiftUpCramped`, similar to `SuperscriptShiftUp` and `SuperscriptShiftUpCramped`, which add a horizontal axis where the primes are placed. There is also a `fixprimes` tweak that we can use to scale and fix the glyph itself. Below, we see how very different the primes from different fonts look (all examples are uniformly scaled), and then examples comparing the original and tweaked primes.

$f'(x) + e^{f'(x)}$ lm original	$f'(x) + e^{f'(x)}$ lm tweaked
$f'(x) + e^{f'(x)}$ lucida original	$f'(x) + e^{f'(x)}$ lucida tweaked
$f'(x) + e^{f'(x)}$ erewhon original	$f'(x) + e^{f'(x)}$ erewhon tweaked
$f'(x) + e^{f'(x)}$ libertinus original	$f'(x) + e^{f'(x)}$ libertinus tweaked

24.8 Font parameters

We add some font parameters, ignore some existing ones, and fix at runtime those that look to be suboptimal. We have no better method than looking at examples, so parameters might be fine-tuned further in the future.

We have already mentioned that we have a few new parameters, `PrimeShiftUp` and `PrimeShiftUpCramped`, to position primes on their own axis, independent of the superscripts. They are also chosen to always be placed outside superscripts, so the inputs `f'^2` and `f^2'` both result in f'^2 . Authors should use parentheses in order to avoid confusion.

$$h^3 + h_2 + h_2^3 + h'$$
$$h^3 + h_2 + h_2^3 + h'$$
$$h^3 + h_2 + h_2^3 + h'$$

Let us briefly mention the other parameters. These are the adapted parameters for T_EX Gyre Bonum:

<code>AccentTopShiftUp</code>	=	-15
<code>FlattenedAccentTopShiftUp</code>	=	-15
<code>AccentBaseDepth</code>	=	50
<code>DelimiterPercent</code>	=	90
<code>DelimiterShortfall</code>	=	400
<code>DisplayOperatorMinHeight</code>	=	1900
<code>SubscriptShiftDown</code>	=	201
<code>SuperscriptShiftUp</code>	=	364
<code>SubscriptShiftDownWithSuperscript</code>	=	"1.4*SubscriptShiftDown"
<code>PrimeShiftUp</code>	=	"1.25*SuperscriptShiftUp"
<code>PrimeShiftUpCramped</code>	=	"1.25*SuperscriptShiftUp"

Some of these are not in OpenType. We can set up much more, but it depends on the font what is needed, and also on user demands.

We have noticed that many font designers seem to have had problems setting some of the values; for example, `DisplayOperatorMinHeight` seems to be off in many fonts.

24.9 Profiling

Let us end with profiling, which is only indirectly related to the tweaking of the fonts. Indeed, font parameters control the vertical positioning of sub- and superscripts. If not carefully set, they might force a non-negative `\lineskip` where not necessary. In the previous section we showed how these parameters were tweaked for Bonum.

Sometimes formulas are too high (or have a too large depth) for the line, and so a `\lineskip` is added so that the lines do not clash. If the lowest part of the top line (typically caused by the depth) and the tallest part of the bottom line (caused by the height) are not close to each other on the line, one might argue that

this `\lineskip` does not have to be added, or at least with reduced amount. This is possible to achieve by adding `\setupalign[profile]`. Let us look at one example.

```

So the question is: how good an approximation to  $\sigma$  is  $\sigma * W\phi$ ? But the attentive reader will
realize that we have already answered this question in the course of proving the sharp Gård-
ing inequality. Indeed, suppose  $\phi \in \mathcal{S}$  is even and  $\|\phi\|_2 = 1$ , and set  $\phi^a(x) = a^{n/4}\phi(a^{1/2}x)$ .
Then we have shown (cf. Remark (2.89)) that  $\sigma - \sigma * W\phi^a \in S_{\rho,\delta}^{m-(\rho-\delta)}$  whenever  $\sigma \in S_{\rho,\delta}^m$ 
is supported in a set where  $\langle \xi \rangle^{\rho+\delta} \approx a$ .

```

No profiling

```

So the question is: how good an approximation to  $\sigma$  is  $\sigma * W\phi$ ? But the attentive reader will
realize that we have already answered this question in the course of proving the sharp Gård-
ing inequality. Indeed, suppose  $\phi \in \mathcal{S}$  is even and  $\|\phi\|_2 = 1$ , and set  $\phi^a(x) = a^{n/4}\phi(a^{1/2}x)$ .
Then we have shown (cf. Remark (2.89)) that  $\sigma - \sigma * W\phi^a \in S_{\rho,\delta}^{m-(\rho-\delta)}$  whenever  $\sigma \in S_{\rho,\delta}^m$ 
is supported in a set where  $\langle \xi \rangle^{\rho+\delta} \approx a$ .

```

Profiling

In the above paragraphs we enabled a helper that shows us where the profiling feature kicks in. We also show the lines (`\showmakeup[line]`). Below we show the example without those helpers. You can judge for yourself which one you prefer.

So the question is: how good an approximation to σ is $\sigma * W\phi$? But the attentive reader will realize that we have already answered this question in the course of proving the sharp Gårding inequality. Indeed, suppose $\phi \in \mathcal{S}$ is even and $\|\phi\|_2 = 1$, and set $\phi^a(x) = a^{n/4}\phi(a^{1/2}x)$. Then we have shown (cf. Remark (2.89)) that $\sigma - \sigma * W\phi^a \in S_{\rho,\delta}^{m-(\rho-\delta)}$ whenever $\sigma \in S_{\rho,\delta}^m$ is supported in a set where $\langle \xi \rangle^{\rho+\delta} \approx a$.

No profiling

So the question is: how good an approximation to σ is $\sigma * W\phi$? But the attentive reader will realize that we have already answered this question in the course of proving the sharp Gårding inequality. Indeed, suppose $\phi \in \mathcal{S}$ is even and $\|\phi\|_2 = 1$, and set $\phi^a(x) = a^{n/4}\phi(a^{1/2}x)$. Then we have shown (cf. Remark (2.89)) that $\sigma - \sigma * W\phi^a \in S_{\rho,\delta}^{m-(\rho-\delta)}$ whenever $\sigma \in S_{\rho,\delta}^m$ is supported in a set where $\langle \xi \rangle^{\rho+\delta} \approx a$.

Profiling

It is worth emphasizing that, contrary to what one might believe at first, the profiling does not substantially affect the compilation time. On a 300-page math book we tried, which usually compiles in about 10 seconds, profiling did not add more than 0.5 seconds. The same observation holds for the other math tweaks we have mentioned: the overhead is negligible.

24.10 Conclusions

All these tweaks can be overloaded per glyph if needed; for some fonts, we indeed do this, in so-called goodie files. The good news is that by doing all this we present the engine with a font that is consistent, which also means that we can more easily control the typeset result in specific circumstances.

The reader may wonder how we ended up with this somewhat confusing state of affairs in the font world. Here are some possible reasons. There is only one reference font, Cambria, and that uses its reference word processor renderer, Word. Then came Xe_LTeX that as far as we know maps OpenType math onto a traditional T_EX engine, so when fonts started coming from the T_EX crowd, traditional dimensions and parameters sort of fit in. When LuaT_EX showed up, it started from the other end: OpenType. That works well with the reference font but less so with that ones coming from T_EX. Eventually more fonts showed up, and it's not clear how

these got tested because some lean towards the traditional and others towards the reference fonts. And, all in all, these fonts mostly seem to be rather untested in real (more complex) math.

The more we looked into the specific properties of OpenType math fonts and rendering, the more we got the feeling that it was some hybrid of what $\text{T}_{\text{E}}\text{X}$ does (with fonts) and ultimately desired behavior. That works well with Cambria and a more or less frozen approach in a word processor, but doesn't suit well with $\text{T}_{\text{E}}\text{X}$. Bits and pieces are missing, which could have been added from the perspective of generalization and imperfections in $\text{T}_{\text{E}}\text{X}$ as well. Lessons learned from decades of dealing with math in macros and math fonts were not reflected in the OpenType fonts and approach, which is of course understandable as OpenType math never especially aimed at $\text{T}_{\text{E}}\text{X}$. But that also means that at some point one has to draw conclusions and make decisions, which is what we do in Con $\text{T}_{\text{E}}\text{X}$ t, LuaMeta $\text{T}_{\text{E}}\text{X}$ and the runtime-adapted fonts. And it gives pretty good and reliable results.

25 Somewhat radical

Here we will discuss an aspect of radicals, namely how variants get applied. Take the following situation:

$$\sqrt{x+1} \sqrt{x-1}$$

$$\sqrt{1+x} \sqrt{1-x}$$

Watch the slight difference in radical heights. Now look at this:

$$\sqrt{\frac{x+1}{x-1}} \quad \sqrt{\frac{1+x}{1-x}}$$

Here we need to make sure that we don't run into the slope, because, when we have a close look at the shapes we see that the radical symbol has a tight bounding box:

$$\sqrt{x+1} \sqrt{x-1}$$

$$\sqrt{1+x} \sqrt{1-x}$$

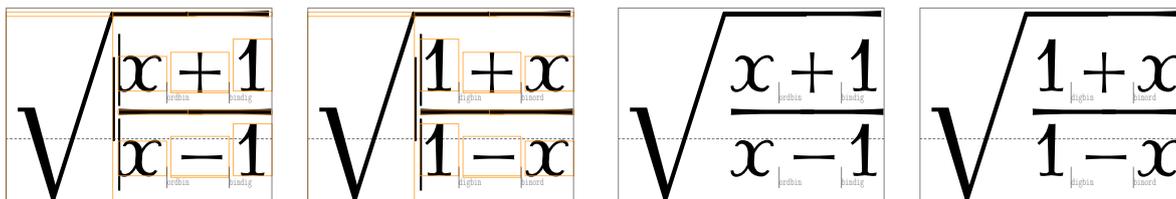
In pagella we get:

$$\sqrt{\frac{x+1}{x-1}} \quad \sqrt{\frac{1+x}{1-x}}$$

and in antykwa:

$$\sqrt{\frac{x+1}{x-1}} \quad \sqrt{\frac{1+x}{1-x}}$$

But now look at this formula:



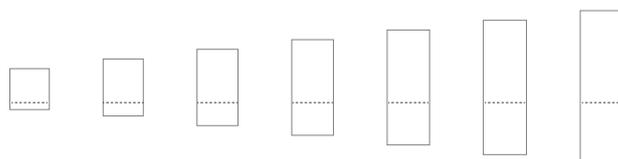
Here we see several mechanisms in action and for a good reason. First of all we want similar subformulas (under the symbol) to have compatible radicals. For this we use special struts so that we always have at least some height. We also compensate for slight differences in depth by setting a minimum depth. Finally we add a bit of margin. That last feature moves the content free from the symbols which means that we can have less distance between the top of the content and the rule. In many fonts that distance is set to a value that prevents clashes and the more slope we have, the more opportunity there is to clash.

When the best fit decision is made for a radical, the effective height of the content (height plus depth of the box) is incremented by a gap variable. The standard specifies the `RadicalVerticalGap` as “Space between the (ink) top of the expression and the bar over it. Suggested: 1.25 default rule thickness.” and `RadicalDisplayStyleVerticalGap` as “Space between the (ink) top of the expression and the bar over it. Suggested: default rule thickness plus .25 times x-height.”. These values are actually rather font dependent because the slope needs to be taken into account; there is also a visual aspect to it.

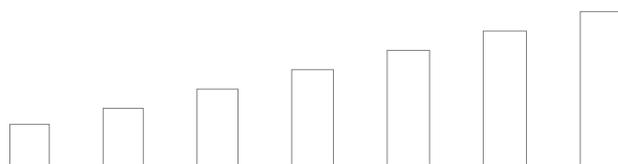
We can't tweak the radical width because the rule has to be attached. If we could we'd have to do it for every variant. So, instead we set up radical like this:

```
\setupmathradical
  [strut=height,      % only height
   leftmargin=.05mq, % fraction of math quad
   mindepth=.05mx]  % fraction of math x height
```

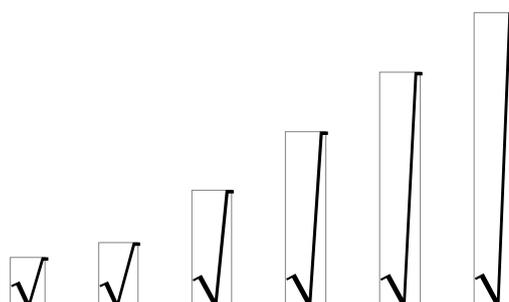
When deciding what size to use, a list of variants is followed till there is a match and when we run out of variants an extensible is constructed. Here is the list of possible sizes in the current font:



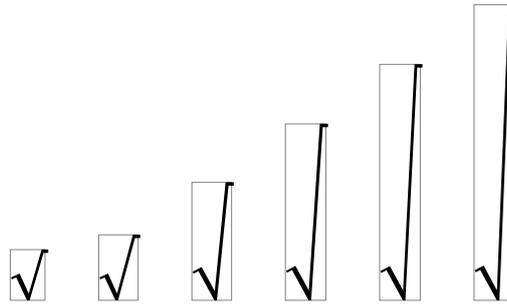
However, when we don't center around the math axis we get a more distinctive view on the steps:



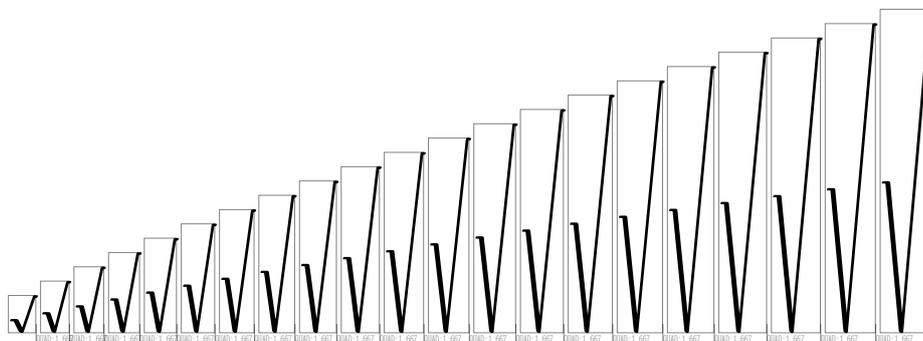
It will be clear that the steps can't be too large but there are fonts out there that behave rather extreme, like Cambria:



The only way out here is to either inject scaled variants into the list of possibilities or to simply ignore all except the first one and go straight to the extensible, so that's what we do, in combination with tweaked parameters and a margin:



As with many font parameters (also in text) one sometimes wonder if font designer test with real examples. There are of course exceptions, for instance the **ebgaramond** font, but that one goes over the top in other areas. Here one can also wonder if the upper half of the range makes sense over an extensible. For consistency one wants steps to be not too small, so that a sequence of radicals looks similar, but steps larger than for instance the height are probably bad.



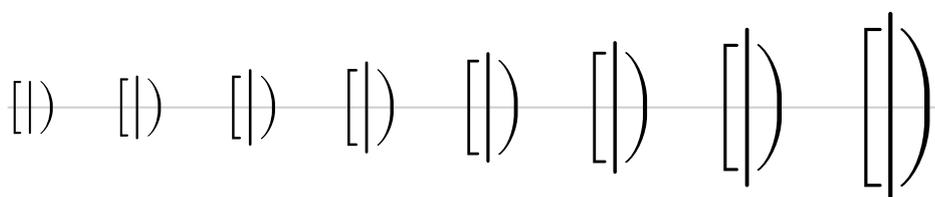
So, as with other examples that we give of tweaking math, it is clear that there is no way around also tweaking radicals, and we're not even talking of the way we fine tune the positioning of degrees in radicals because that is also a neglected area in OpenType math fonts.

26 Between bars

Inconsistencies

The bar in math can be a real pain. There are several reasons for this, for instance there is no proper left, middle and right bar in Unicode and as a result there is more work involved in getting them spaced well. Another possible issue can be to make them fit well with other fences. You expect the bars in $\left|$ and $\right|$ to look similar.

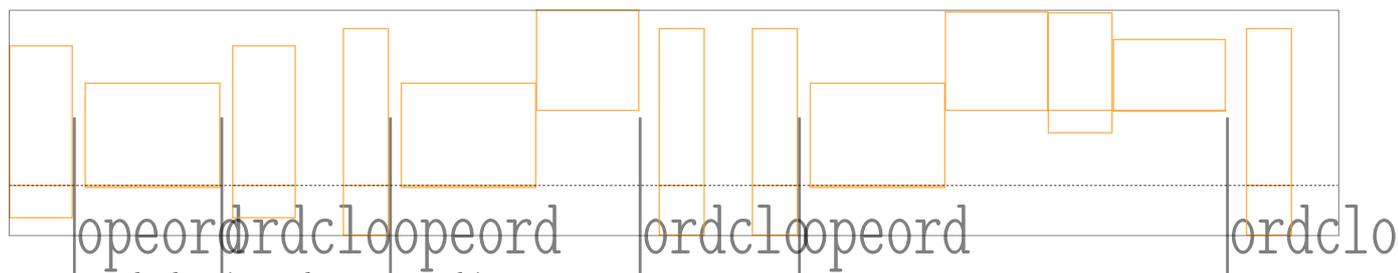
However, math fonts have their surprises:



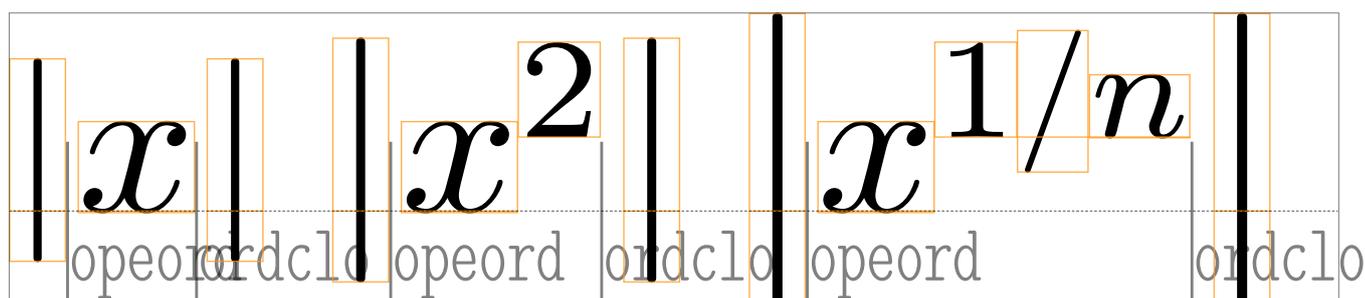
In Latin Modern only the first variant is tuned to work together but for larger sizes the bars stick out. This is a problem when we want fences to adapt. The fact that such side effects probably get unnoticed comes from the fact that macro packages assume `\bigg` and friends to be used but in ConTeXt, and especially LMTX, we have various mechanisms for this. One method is based on selecting specific variants, in the case of Latin Modern 1, 4, 6 and 7, where in fact 7 is the last one before we switch to extensible fences. One can try to use a different selection for brackets and bars when there is no nice match but there are no equal height matches.

```
\im {\left| x          \right|}  
\im {\left| x^2       \right|}  
\im {\left| x^{1/n}   \right|}
```

These example formulas can trigger a larger fence:

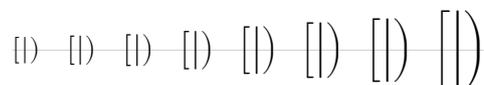


In untweaked Latin Modern we get this:

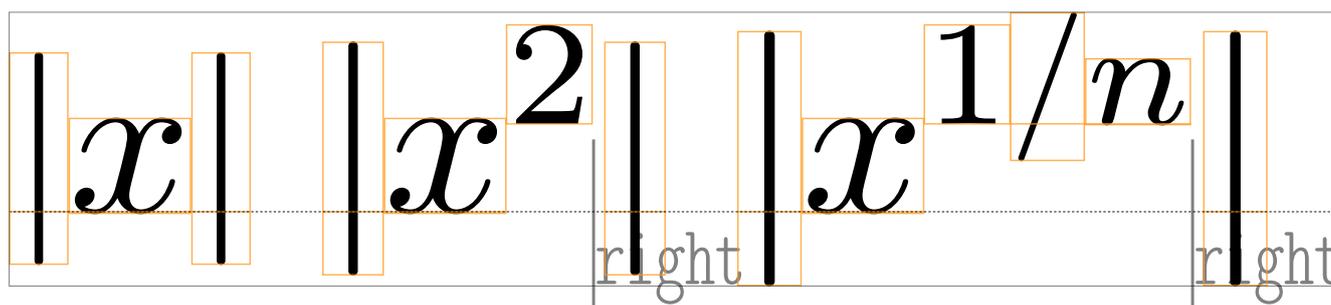


The slash in this font is rather high and therefore triggers the larger fence. One can configure this with the `\delimiterfactor` and `\delimitershortfall` but as you can see values have to relate to the font. In LMTX we set them to 1000 and 0pt and use the LuaMetaTeX equivalent font variables instead, so we can indeed fine tune per font.

To come back to the mismatch in fences, this is dealt with in a tweak: we scale the single, double and triple bars to match the brackets:



Combined with proper settings for the factor (or percentage in OpenType math speak) and shortfall, we now get:



When we let the upgraded math subsystem evolve we make many examples. Unfortunately there is always an exception. For instance, we test a specific font, notice something, deal with it, even test all fonts in inline and display math and then after months the exception shows up. In this case it was the `right` in a superscript that (only?) in Latin Modern goes over the top. Actually we had noticed that bars are often inconsistent so we had a `fixbars` tweak. However, for Latin Modern we found that the inconsistency between bars and other fences needed something more drastic. Of course fixing the font is best but we're beyond that stage now: the fonts are basically frozen.

A close inspection of the too large fence which itself results from it being larger than expected by design (which we noticed by adding parentheses) itself was the result from deciding to configure additional inter-atom spacing for open and close fences (see below) which then brings us back to the fact that one bar serves three purposes. We might actually introduce these three (left, middle and right) at some point.

$$\left| \frac{n^3 - 2n + 1}{n^5 - 3} - 0 \right| < \frac{4}{n^2}$$

Missing shapes

The tweak discussed in the previous section is a brute force one: we put an extensible on the base glyph. Among the arguments for doing this is that we want to be able to add consistent double and triple bars. Without mentioning fonts explicitly (as some might get fixed after we file bug reports) this is what we observed:

- There are single bars, double bars and triple bars and each has variants and extensibles. This is okay.
- Most are there but the triple bar has no variants and extensibles
- We have all three base characters but no variants. The extensible has a different width.
- Single, double and triple bars are inconsistent with each other.
- Everything is there but widths differ per variant; some match the parenthesis, brackets and braces but not consistently.

- The different variant sizes are out of sync with the sizes of parenthesis etc. and this makes for inconsistent matches, especially when also the width and positioning differs.
- Spacing between and around double and triple bars isn't always consistent.

These observations lead us to the conclusion that there is no single tweak that can fix this. Adapting the ‘ad-dbars’ tweak to deal with all this made for too many alternatives in checks and fixes to feel comfortable with. This is why we decided to come up with companion fonts that provide the missing double and triple variants and extensibles consistent with the single ones, fix spacing in double and triple ones, fix inconsistent widths of bars, etc. Minor details like bad positioning are already handled well so we can keep the ‘design’ as it is.



Different sizes

middle

